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QUESTIONS AND EXERCISES

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IN

ELEMENTARY MATHEMATICS

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ADAPTED TO

MATRICULATION, RESPONSIONS, FIRST PUBLIC EXAMINATION, AND THE OXFORD AND CAMBRIDGE LOCAL AND CERTIFICATE EXAMINATIONS

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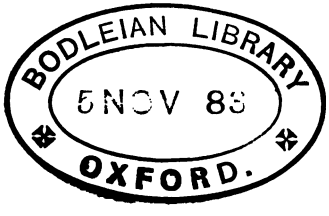

PREFACE

TO

THE SECOND EDITION.

THE OBJECT of this volume is to furnish candidates for examination in Elementary Mathematics with examples for practice and test papers in Arithmetic and Algebra, and with suggestive questions in Euclid.

The First Edition was limited to the subjects required at Matriculation and in Responsions. The Second Edition has been extended so as to meet the requirements of the First Public Examination, and of the Oxford and Cambridge Local and Certificate Examinations. With this view new matter has been introduced, and, in order to avoid increasing the bulk of the volume, some of the easier Questions and earlier Examination Papers have been omitted.

All the Examination Papers in Arithmetic, except the last one, and those in Algebra I. to XXVIII., are substantially the same as papers which have been set in

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In its present form the book will be found useful in preparation for any examination which includes the subjects of Arithmetic, Algebra to Quadratic Equations with Proportion, and Euclid Books I. to IV.

OXFORD: *June* 1877.

THIRD EDITION.

Five more papers in Algebra have been added, adapted to Moderations and the Certificate and Senior Local Examinations.

OXFORD: *November* 1879.

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MATHEMATICAL
QUESTIONS AND EXERCISES.

ARITHMETICAL.

I. TABLES AND REDUCTION.

1. Write out the Tables of

Avoirdupois Weight.	Long Measure.
Troy Weight.	Square Measure.
Apothecaries' Weight.	Solid Measure.
Measure of Capacity.	

2. What are the different uses to which these Tables are severally applied?

3. State the value in grains of the lb. and oz. in each of the Tables of Weight.

4. Express 56 lbs. Av. in lbs. &c. Troy; and 5 oz. 9 dwts. 9 grns. Troy in Av.

5. Reduce

(1) 2 miles 3 furlongs 20 poles 1 yard to inches.
(2) 1 furlong 15 poles 1 foot 6 inches to yards.
(3) 357000 inches to miles, furlongs, poles, &c.
(4) 5763 feet to miles, furlongs, poles, &c.

6. Reduce

(1) 5 acres 3 roods 35 poles to square inches.

(2) 5 poles 6 feet 108 inches (square measure) to yards.

(3) 3 poles 7 yards 2 feet 36 inches (square measure) to yards.

(4) 60500 square inches to poles, yards, &c.

7. What is a quarter of wheat? a pipe of wine? an ell of cloth? a firkin of ale? a ream of paper?

8. Give the value of a groat, a mark, a moidore, a franc, an American dollar, a Prussian thaler.

9. If a mina = 4*l.* 1*s.* 3*d.*, reduce 568*l.* 15*s.* to minæ.

II. THE FIRST FOUR RULES.

1. Add together 4 tons 3 qrs. 20 lbs. 9 oz. 12 drs. and 3 tons 2 qrs. 18 lbs. 10 oz. 10 drs.

2. Subtract 2 miles 6 furlongs 4 poles and 2 inches from 4 miles and 8 yards.

3. Multiply 3 tons 12 cwt. 1 qr. 17 lbs. by 11.

4. What do 12 dozen chains weigh, if each contains 2 oz. 9 dwts. 3 grs. of metal?

5. What is the price of 32 yards of ribbon at 1*s.* $4\frac{3}{4}$*d.* per yard?

6. A person wastes three-quarters of an hour every day: how many days will he have wasted between to-day and the end of the year?

7. If every 1*d.* in the £ of income tax adds 1000000*l.* to the revenue, what is the total amount of income on which the tax is levied?

8. *In a 'bumping' race the second boat gains 5 inches on the first at every stroke, and bumps it after 156*

strokes: what was the distance between them at starting?

9. Divide

(1) 23*l.* 15*s.* 7¾*d.* by 37.
(2) 885*l.* 19*s.* 3¾*d.* by 327.
(3) 572 yards 3 quarters 1 nail by 47.
(4) 1 acre 2 roods 32 perches 3 yards 0 feet 78 inches by 7.

10. If a person spends 278*l.* 6*s.* 3*d.* a year, how much does he spend each day (1) in an ordinary year, (2) in a leap year?

11. If a man spends as much in four months as he gains in three, how much will he lay by in a year from an income of 150*l.*?

12. A dozen persons hire an omnibus at 1*l.* 17*s.* 6*d.* for the day: how much more must each pay than he would if the party were 15 in number?

13. At the Handel festival 23760 persons entered the Crystal Palace between 10 A.M. and 4 P.M.: what was the average per minute?

14. A ship left England at noon on January 1, 1851; she reached a port in Australia at midnight (Greenwich time) on April 23: supposing the length of the voyage to be 18225 miles, at what rate per hour did she run?

15. If 48 sovereigns are coined out of 16 ounces of metal, of which one-twentieth part is alloy, how much alloy is there in each sovereign?

16. Divide 21*l.* 10*s.* among three persons, so that one may have 3*l.* 10*s.* more than each of the others.

17. How many half-crowns, how many sixpences, and how many fourpences are there in 25*l.*?

18. *How many* rails, each weighing 1 qr. 3 lbs., can be *made out of* 3 tons 16 cwt. 1 qr. 16 lbs. of iron?

19. How many plots of $2\frac{1}{2}$ acres each can be made out of an estate a mile square?

20. A man advances 2 feet 8 inches each step: how many steps will he take in 2 miles?

21. What would be the amount of a person's income, if the tax on it at *7d.* in the pound amounted to *26l.* 10*s.* 10*d.*?

22. A packet of sugar worth 8*d.* per lb. cost 1*l.* 8*s.* $10\frac{1}{2}$*d.*: what was the quantity?

23. Divide *20l. 2s. 6d.* into two sums of money, one of which shall contain as many half-crowns as the other contains shillings.

24. A merchant spends *1636l.* 5*s.* on equal quantities of wheat at 42*s.*, barley at 21*s.*, and oats at 14*s.* a quarter: what quantity of each will he have?

25. What is meant by Quotient, Sum, Product, Factor, Integer?

26. What is the test of a number being divisible by 2, 3, 4, 5, 6, 8, 9, 11?

III. GREATEST COMMON MEASURE AND LEAST COMMON MULTIPLE.

1. Find the greatest common measure of

(1) 72 and 192.
(2) 429 and 15.
(3) 3775 and 1000.
(4) 203 and 2291.
(5) 2163 and 504.
(6) 5292 and 1520.
(7) 3575 and 4719.
(8) 5292 and 8316.
(9) 731, 2329, and 3502.
(10) 3240, 1224, and 2068.
(11) 1125, 1575, and 3150.
(12) 286720 and 1310309.

2. Find the least common multiple of

(1) 12, 21, 28, 30, 35. (2) 18, 16, 63, 24, 56.
(3) 4, 9, 16, 28, 42. (4) 6, 15, 18, 35, 72.
(5) 5, 7, 9, 11, 15. (6) 1, 2, 3, 4, 5, 6, 7, 8, 9.
(7) 25, 27, 9, 11, 15. (8) 8, 12, 18, 24, 27.
(9) 3, 7, 19, 42, 114. (10) 3, 8, 17, 40, 153.
(11) 945, 990. (12) 1530, 1581.

3. Find the greatest common measure and least common multiple of

(1) 28431 and 216. (2) 24720 and 4155.
(3) 9450 and 58905. (4) 1919 and 3857.
(5) 125, 175, 225. (6) 144, 176, 272.
(7) 161, 253, 299.

4. Define the terms Greatest Common Measure and Least Common Multiple.

IV. FRACTIONS (PURE).

1. Reduce to its lowest terms

(1) $\frac{252}{364}$. (2) $\frac{1344}{1536}$. (3) $\frac{71091}{80037}$. (4) $\frac{35715}{47119}$.
(5) $\frac{9193}{20677}$. (6) $\frac{4155}{24720}$. (7) $\frac{29393}{83398}$. (8) $\frac{9019}{96721}$.
(9) $\frac{195297}{236067}$.

2. Reduce the compound fractions

(1) $\frac{3}{11}$ of $1\frac{2}{9}$ of $5\frac{1}{3}$ of $\frac{1}{4}$. (2) $\frac{3}{8}$ of $6\frac{1}{2}$ of $\frac{13}{14}$ of $\frac{7}{26}$.
(3) $\frac{5}{8}$ of $\frac{3}{7}$ of $\frac{1}{2}$ of $\frac{14}{15}$. (4) $3\frac{1}{7}$ of $2\frac{13}{25}$ of $\frac{5}{11}$.
(5) $1\frac{1}{8}$ of $1\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{9}$ of 12.

3. Multiply $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{9}{8}$ by $\frac{1}{7}$ of $8\frac{2}{3}$, and find the value of $2\frac{1}{5} \times 1\frac{4}{6}$ of $1\frac{2}{13} \times 3\frac{1}{4}$ of $1\frac{5}{11}$.

4. Divide

(1) $\frac{3}{4}$ by $\frac{5}{6}$. (2) $\frac{9}{11}$ by $\frac{3}{4}$. (3) $\frac{1}{2}$ by $\frac{1}{3}$.
(4) $\frac{4}{15}$ by $\frac{2}{5}$. (5) $\frac{2}{7}$ by $\frac{7}{3}$. (6) $5\frac{1}{4}$ by $\frac{2}{5}$.
(7) $20\frac{5}{14}$ by $17\frac{1}{12}$. (8) $\frac{2}{3}$ of $\frac{1}{3}$ by $\frac{5}{7}$ of $\frac{3}{5}$.
(9) $\frac{5}{14}$ of $5\frac{1}{4}$ of $\frac{18}{16}\frac{5}{}$ by $1\frac{10}{27}$

5. Simplify

(1) $\dfrac{1}{3 \times \frac{3}{4}}$. (2) $\dfrac{9\frac{7}{9}}{2\frac{1}{27}}$ (3) $\dfrac{5\frac{3}{11}}{2\frac{1}{11}}$. (4) $\dfrac{8\frac{3}{4}}{5\frac{5}{8}}$.

(5) $\dfrac{15\frac{3}{5}}{7\frac{4}{5}}$. (6) $\dfrac{3\frac{3}{7} \text{ of } 2\frac{11}{12}}{\frac{1}{33} \text{ of } 8\frac{9}{14}}$. (7) $\dfrac{4\frac{1}{2} \text{ of } \frac{5}{27}}{5\frac{3}{5} \text{ of } 1\frac{3}{9}}$.

(8) $\dfrac{3\frac{3}{4}}{5} \times \dfrac{\frac{11}{12}}{7\frac{1}{18}}$. (9) $\dfrac{12}{4\frac{1}{5}} \times \dfrac{\frac{4}{5}}{\frac{2}{3}}$.

(10) $\dfrac{3\frac{1}{3} \text{ of } 2\frac{2}{5}}{1\frac{3}{4} \text{ of } 1\frac{5}{7}} \div 1\frac{1}{2}$. (11) $3\frac{1}{5}$ of $6\frac{1}{4} \div \dfrac{3}{1\frac{1}{5}}$.

(12) $7\frac{1}{3}$ of $2\frac{5}{11} \div (\frac{9}{19}$ of $16\frac{5}{8})$.

(13) $3\frac{1}{7}$ of $2\frac{13}{25}$ of $\frac{5}{11} \times \dfrac{\frac{5}{9}}{1\frac{1}{3}} \div \dfrac{1\frac{1}{8}}{\frac{3}{5}}$.

6. Add together

(1) $\frac{2}{5}$, $\frac{1}{6}$, $\frac{7}{3}$. (2) $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{10}$, $\frac{8}{15}$.
(3) $2\frac{2}{3}$, $4\frac{1}{6}$, $3\frac{5}{12}$. (4) $3\frac{1}{2}$ of $1\frac{1}{5}$, $2\frac{2}{5}$ of $1\frac{1}{6}$.
(5) $2\frac{1}{5}$ of $7\frac{1}{2}$, $\dfrac{5\frac{1}{4}}{1\frac{1}{8}}$, $\frac{5}{21}$ of $6\frac{1}{3}$, $\dfrac{\frac{5}{9}}{3\frac{1}{2}}$.

7. Subtract

(1) $\frac{3}{4}$ from $\frac{8}{9}$. (2) $\frac{1}{2}$ of $\frac{1}{3}$ from $\frac{3}{4}$ of $\frac{4}{5}$.
(3) $\frac{2}{3}$ of $1\frac{1}{2}$ from $\frac{1}{2}$ of $2\frac{2}{3}$.
(4) $12\frac{2}{3}$ from $14\frac{3}{4}$. (5) $121\frac{5}{6}$ from $202\frac{1}{2}$.

8. Simplify

(1) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{5} - \frac{1}{6}$. (2) $\frac{1}{2} - \frac{4}{9} + \frac{5}{13} - \frac{7}{16}$.
(3) $2\frac{1}{2} + \frac{3}{4}$ of $5\frac{1}{2} + 4\frac{7}{11} + 5$.
(4) $1\frac{13}{8} + \frac{3}{5}$ of $\frac{10}{9} - \frac{11}{15}$ of $6\frac{1}{4}$.
(5) $\frac{3}{5} + 1\frac{7}{10} \times \dfrac{4}{3\frac{1}{5}}$. (6) $1\frac{1}{3} - (1\frac{1}{3}$ of $\frac{1}{2})$.

(7) $\frac{1}{3} + 6\frac{1}{2} \times \frac{3\frac{4}{9}}{7} - \frac{1}{3} \times \frac{2}{5}$.

(8) $\frac{\frac{3}{5}}{\frac{8}{9}} + \frac{\frac{5}{8}}{2\frac{1}{3} \times 1\frac{1}{3}} \times \frac{1}{80}$. (9) $\frac{6\frac{5}{7} - (6 \times \frac{2}{7})}{5}$.

(10) $\frac{1\frac{1}{4} \text{ of } \frac{6}{15}}{\frac{2}{17} + \frac{11}{51}}$. (11) $\frac{2\frac{1}{4} - 1\frac{3}{5}}{8\frac{1}{2} + 1\frac{9}{10}}$.

(12) $\frac{10 \times \frac{1}{7}}{3 + \frac{2}{7}}$. (13) $\frac{23}{2\frac{2}{3} + \frac{2}{5}}$.

(14) $(2\frac{1}{2} + \frac{1}{6}) \div (2\frac{2}{3} - \frac{1}{8})$.

(15) $2\frac{5}{63} + (\frac{1}{7} \times \frac{4}{9}) - 2\frac{1}{2} \text{ of } (\frac{2}{3} \div \frac{7}{9})$.

(16) $1\frac{28}{36} + (\frac{1}{6} \times \frac{7}{12}) - \frac{1}{2} \text{ of } (\frac{3}{4} \div \frac{2}{3})$.

(17) $\frac{1\frac{1}{2}}{1\frac{1}{4}} \div \left(\frac{3\frac{7}{9}}{2\frac{4}{5}} \times \frac{5}{6}\right) + \frac{7}{16}$.

(18) $\cfrac{2}{3 + \cfrac{4}{5 + \frac{6}{7}}}$. (19) $\cfrac{1}{2 + \cfrac{1}{2 + \frac{1}{3}}}$.

(20) $\cfrac{1}{2 + \cfrac{3}{4 + \cfrac{5}{6 + \frac{7}{8}}}}$

(21) $\frac{3}{14} \times \frac{5\frac{5}{6} - 1\frac{5}{18}}{\frac{1}{9} \text{ of } \frac{111}{2}} \div \frac{\frac{2}{7}\left(\frac{58}{3} + 21\frac{2}{3}\right)}{\frac{1}{1 + \frac{3}{8}} + \frac{12}{13} \text{ of } \frac{91}{14}}$.

9. Add together $\frac{5}{6}$ and $2\frac{3}{5}$, and subtract their sum from 4.

10. Subtract $\frac{5}{6}$ from $\frac{6}{7}$, $\frac{6}{7}$ from $\frac{7}{8}$, and $\frac{7}{8}$ from $1\frac{1}{5}$, and add together the results.

11. Which is the greater, $\frac{1}{3}$ of 4 or $\frac{1}{4}$ of 5, and by how much?

12. Which is the greater, $\frac{2}{3}$ of $1\frac{1}{5}$, or $\frac{1}{7}$ of $5\frac{1}{6}$, and by *how much?*

13. What is the value of $(\frac{3}{7} \times \frac{9}{14}) \times \frac{2}{15}$, and what will be the remainder when $\frac{3}{90}$ is subtracted from the product so obtained?

14. Multiply $9 - 3\frac{6}{25}$ by $16\frac{2}{3}$ of $1\frac{1}{12}$.

15. Divide $5\frac{5}{18} - \frac{43}{36}$ by $1\frac{2}{7}$ of $8\frac{1}{6}$.

16. Find the value of $\frac{2\frac{3}{4} \text{ of } 3\frac{2}{3}}{2\frac{5}{6} + \frac{1}{2} + \frac{1}{3}}$ multiplied by $\frac{2}{3}$ of 6.

17. Explain the principles by which the addition and subtraction of fractions is effected.

18. Prove the rule for division of vulgar fractions by each other.

19. Define the terms fraction, proper fraction, improper fraction, mixed number, compound fraction, complex fraction.

V. FRACTIONS (MIXED).

1. Find the value of

(1) $\frac{11}{8}$ of 1*l.* (2) $\frac{7}{12}$ of a guinea. (3) $3\frac{3}{4}$ of 1*l.*

(4) $2\frac{1}{2}$ of half a guinea. (5) $\frac{3}{5}$ of 3*l.* 11*s.* 3*d.*

(6) $\frac{1}{3}$ of $3\frac{1}{4}$ of $\frac{12}{13}$ of 1*l.*

(7) $2\frac{1}{2}$ of $5\frac{1}{9}$ of 15*l.* 19*s.* 6*d.* (8) $\frac{5}{16}$ of 16*s.* $4\frac{4}{5}$*d.*

(9) $\frac{2}{5}$ of 4*s.* 10*d.* (10) $\frac{2\frac{4}{9}}{17\frac{3}{5}}$ of a yard.

(11) $\frac{2}{15}$ of a furlong. (12) $\frac{20}{33}$ of a mile.

(13) $\frac{2}{3}$ of $\frac{6}{5}$ of $3\frac{1}{2}$ miles.

(14) $3\frac{3}{8}$ of $2\frac{1}{9}$ square yards. (15) $\frac{5}{8}$ of an acre.

(16) $\frac{33}{114}$ of a bushel. (17) $\frac{5}{64}$ of 1 ton.

(18) $4\frac{1}{6}$*l.* + $11\frac{1}{3}$*s.* + $7\frac{9}{12}$*d.*

(19) $\frac{2}{3}$ of 5 guineas − $\frac{3}{4}$ of $\frac{7}{9}$*l.*

(20) $2\frac{2}{3}$ *of* 5 *bushels* + $1\frac{1}{3}$ of 5 gallons.

(21) $\{1\frac{1}{7}$ *of* $\frac{5}{9}$ *of* $1\frac{2}{5} - (1 - \frac{1}{3} + \frac{1}{9})\}$ of 1s. 6d.

(22) $\frac{\frac{1}{7}}{4\frac{1}{3} + 3\frac{2}{3}} \times \frac{1}{2}$ of 1 cwt.

(23) $\left[\frac{5\frac{1}{4}}{2\frac{4}{5}} + \frac{185}{216} \div 1\frac{10}{27} - \{2\frac{1}{2} - (1\frac{1}{4} - \frac{3}{8})\}\right]$ of 2 lbs. 9 oz. 15 dwts.

2. Multiply

(1) 38*l.* 6*s.* $9\frac{1}{4}d.$ by $15\frac{1}{5}$.
(2) 30*l.* 14*s.* $6\frac{1}{2}d.$ by $\frac{4}{23}$.
(3) 59*l.* 18*s.* $10\frac{3}{4}d.$ by $\frac{7}{100}$.
(4) 2 yds. 1 ft. $7\frac{4}{5}$ in. by $\frac{11}{18}$.

3. Divide

(1) 125*l.* 4*s.* $2\frac{1}{4}d.$ by $5\frac{1}{2}$.
(2) 2 weeks 3 days by $3\frac{3}{4}$.
(3) 59*l.* 12*s.* $0\frac{3}{4}d.$ by $12\frac{1}{12}$.
(4) 2 tons $6\frac{5}{7}$ cwts. by $8\frac{5}{10}$.

4. What is the value of the sum of $\frac{16}{18}$ of a guinea, $\frac{7}{30}$ of a sovereign, and $\frac{23}{11}$ of a crown?

5. If $\frac{7}{12}$ of an estate be worth 3003*l.*, what is the whole worth?

6. If I sell $\frac{5}{8}$ of $\frac{4}{9}$ of an estate, what portion of it have I left?

7. If a person pays away $\frac{2}{3}$ of his income, and invests $\frac{3}{5}$ of the remainder, what portion of the whole has he left?

8. A person, having sold $\frac{2}{3}$ of an estate, bequeaths $\frac{3}{4}$ of the remainder to his son, and leaves the rest to be divided equally between three charities: if each of the latter receives 136*l.* 5*s.* what was the value of the whole estate?

9. Two pipes together fill a cistern in 1 hour, and one of them alone fills it in $1\frac{1}{2}$ hour: how long will it take the other alone to fill it?

10. If one man can do a piece of work in 10 days which would take another man 15 days, in what time could they do it both working together?

11. A and B can do a piece of work in 7 days which B and C can do in 8 days, or all three together can do in 5 days: what portion of the whole can each do in one day?

12. Reduce

(1) 3 cwt. 2 qrs. 3 lbs. to the fraction of a ton.
(2) 1 rood 5 perches to the fraction of an acre.
(3) 12 gallons 2 quarts of wine to the fraction of a hogshead.
(4) 5 furlongs 13 poles 1 yard 2 feet 6 inches to the fraction of a mile.
(5) 2 weeks 2 days $19\frac{1}{8}$ hours to the fraction of a month.
(6) 12*s.* 9*d.* to the fraction of 15*s.* 6*d.*
(7) 8 cwt. 2 qrs. 2 lbs. to the fraction of 18 cwt. 3 qrs. 5 lbs.
(8) 14 hours 15 minutes to the fraction of $3\frac{1}{2}$ days.
(9) 6*s.* $8\frac{1}{2}$*d.* to the fraction of $1\frac{3}{4}$*d.*
(10) $2\frac{1}{2}$ guineas to the fraction of $1\frac{3}{4}$*l.*
(11) $\frac{2}{15}$ of a guinea + $\frac{1}{8}$ of a sixpence to the fraction of 1*l.*
(12) $2\frac{1}{2}$ of a groat + $\frac{5}{6}$ of a crown + $\frac{1}{4}$ of a guinea to the fraction of 1*l.*

13. Reduce

(1) $\frac{4}{7}$ of a guinea to the fraction of a pound.
(2) $\frac{2}{15}$*l.* to the fraction of a penny.
(3) $\frac{4}{7}$ of a guinea to the fraction of 23*s.* 4*d.*
(4) $\frac{1}{2}$ of $\frac{5}{3}$ of $\frac{1}{5}$ of 15*l.* to the fraction of 2*s.* 8*d.*
(5) $\frac{2}{7}$ *cwt. to the* fraction of a lb.
(6) $\frac{5}{8}$ *of lb. Troy* to the fraction of lb. Avoir.

(7) $\frac{4}{784}$ of a cwt. to the fraction of a lb.
(8) $2\frac{5}{9}$ of a week to the fraction of an hour.
(9) $\frac{2}{17}$ of 15*s.* 6*d.* to the fraction of 5*l.*
(10) $\frac{9}{13}$ of a guinea to the fraction of a sovereign.
(11) $\frac{8}{11}$ of a pint to the fraction of a gallon.

VI. DECIMALS (PURE).

1. Add together

(1) 2·03, 30·2. (2) 8·234, ·715, 1·151.
(3) ·0023, 2·36, 250, ·527.

2. Substract

(1) 2·03 from 20·2. (2) ·005 from ·05.
(3) ·075 from 57.

3. Find the value of 60·6 + ·005 − 6·0606.

4. Multiply

(1) 3·26 by 1·02. (2) 3·25 by 2·04.
(3) 375 by ·08. (4) 1·23 by ·0012.
(5) 32·719 by ·00854. (6) ·2001 by 200·001.
(7) 3025 by ·000015. (8) 2·048 by ·0008.
(9) ·111 by ·011. (10) ·036 by ·05.
(11) ·000476 by ·0078. (12) ·7854 by $4\frac{1}{2}$.
(13) ·0004 by 250. (14) ·0001 by 10000.
(15) ·026 by 3·3, by ·33, by 33, and by ·033 separately.
(16) 2·23 by ·23, by 2·3, by ·023, and by 23 separately.

5. Divide

(1) 998·824392 by ·018. (2) 160·68 by 3·9.
(3) ·3964 by ·75. (4) 17·1668 by ·0035.

(5) 52 by 3·2. (6) 1·8 by 500. (7) 611 by ·13.
(8) ·07306 by 1·405. (9) 119·18 by ·059.
(10) ·85 by ·00017. (11) 6·8 by ·000017.
(12) 2·961 by ·007. (13) ·03 by ·001.
(14) 3 by ·01. (15) 2175·68 by 100.
(16) 736 by ·02 and ·736 by 2.
(17) 101 by 1·01, and ·101 by 10·1.
(18) 48·6 by 1·8, and by 18; and 4·86 by ·18, by 1·8, and by ·0018.
(19) 3·52 by 2·2, by ·22, and by 22; and 35·2 by ·22, and by ·022.
(20) 6978·6 by 3·24. (21) 76 by ·1254.
(22) ·8727587 by ·162. (23) ·05625 by ·0275.
(24) 76 by ·01254. (25) ·2 by ·021.
(26) ·0002 by ·0163. (27) $2\frac{1}{12}$ by ·15.

6. Which is the greatest and which the least of
$2{\cdot}5 \times {\cdot}075$, $2{\cdot}625 \div 7$, $5 \times {\cdot}05$?

7. Find the value of $\dfrac{{\cdot}375 \times {\cdot}375 - {\cdot}025 \times {\cdot}025}{{\cdot}375 - {\cdot}025}$.

8. Reduce to decimals

(1) $\frac{4}{5}$. (2) $\frac{36}{50}$. (3) $\frac{7}{250}$. (4) $\frac{1062}{125}$. (5) $1\frac{1}{4}$.
(6) $3\frac{5}{8}$. (7) $\frac{1}{25}$ of $\frac{1}{2}$. (8) $\frac{5}{6}$. (9) $\frac{1}{18}$.
(10) $\frac{16}{81}$. (11) $\frac{59}{180}$. (12) $\frac{3}{11}$. (13) $\frac{1}{33}$.
(14) $\frac{129}{55}$. (15) $\frac{1}{37}$. (16) $\frac{2929}{16653}$.
(17) $17\frac{13}{700}$. (18) $\frac{11}{14}$ of $\frac{11}{13}$.

9. Find the decimals equivalent to $\frac{3}{5} + \frac{7}{2} - \frac{1}{9}$, and to

$$\frac{2\frac{3}{4}}{4\frac{2}{7}} - \frac{3\frac{2}{5}}{4\frac{1}{4}} + \frac{\frac{4}{7}}{2\frac{1}{2}}$$

10. Find the value of $13\frac{1}{4} + 1\frac{2}{5} + 8\frac{3}{125} - 4\frac{1}{8}$ by decimals and by vulgar fractions.

11. When a fraction has been reduced to its lowest *terms, and it is required* to express it as a decimal, how

can we tell whether that decimal will be finite or a circulator? Apply the rule to $\frac{3}{64}$ and to $\frac{41}{14}$.

12. Reduce to vulgar fractions in their lowest terms

(1) ·138. (2) ·305. (3) ·0075. (4) ·00029.
(5) ·26125. (6) ·0031625. (7) 3·6. (8) 5·68.
(9) 3·625 + ·064 − ·0008. (10) $\cdot\dot{3}\dot{6}$
(11) $\cdot 0\dot{3}$ (12) $\cdot\dot{9}$ (13) $\cdot\dot{9}7\dot{2}$.
(14) $\cdot\dot{7}14285\dot{7}$. (15) $\cdot 50\dot{9}$. (16) $\cdot 053\dot{6}$.
(17) $5\cdot 0\dot{3}\dot{6}$. (18) $\cdot 01728\dot{8}$. (19) $1\cdot 14\dot{5}$.
(20) $1\cdot 2\dot{1}42857\dot{1}$. (21) $\cdot 5357142\dot{8}$.
(22) $\cdot 02\dot{1}\dot{8}$. (23) $2\cdot 00\dot{1}\dot{8}$. (24) 0·6355 &c.
(25) 10·01001001 &c.

(26) $$\cfrac{2}{3+\cfrac{\cdot 2}{\cdot 02+\cfrac{\cdot 02}{3+\cfrac{\cdot 02}{3\cdot 002}}}}.$$

13. Add together 23·54, 156·213, 2·0004, ·1245; and also 5·127 and 2·4004.

14. From 8·02362 take ·849, from 3·16 take 2·059, and from ·3 take ·03.

15. Add the following correctly to 5 decimal places:
1·05, ·52, 25·00064, ·24123, ·1234.

16. Multiply and divide 2631·874 by 70.

17. Divide ·036 by ·03, and 2·345 by ·003.

18. Find the sum, difference, product, and quotient of ·025 and ·03.

19. State and prove the rule for reducing circulating decimals to vulgar fractions.

20. What is a decimal? Explain the effect of moving the decimal point one place to the right or left.

21. What is the effect of adding cyphers, 1st to the right of a decimal, 2nd to the left of it? Prove your answers.

VII. DECIMALS MIXED.

1. Reduce

(1) 3 farthings to the decimal of 1*l.*
(2) 7*s.* 9*d.* to the decimal of 1*l.*
(3) 9*s.* 8¾*d.* to the decimal of 1*l.*
(4) 1*l.* 2*s.* 6*d.* to the decimal of 1*l.*
(5) 2 qrs. 14 lbs. to the decimal of a cwt.
(6) 3 cwt. 14 lbs. to the decimal of a ton.
(7) 3 quarts 1 pint to the decimal of a gallon.
(8) 1 mile 2 furlongs to the decimal of a league.
(9) 6 furlongs to the decimal of a league.
(10) ·056 of a pole to the decimal of an acre.
(11) 8*s.* 4*d.* to the decimal of 1*l.*
(12) 7*s.* 8*d.* to the decimal of 1*l.*
(13) 17*s.* 6½*d.* to the decimal of 1*l.*
(14) 4*s.* 4*d.* to the decimal of a guinea.
(15) 3*l.* 15*s.* 6½*d.* to the decimal of 4*l.*
(16) 2*l.* 13*s.* 4*d.* to the decimal of 4*l.* 5*s.*
(17) 2*l.* 15*s.* 9*d.* to the decimal of 3*s.* 8½*d.*
(18) 14 minutes to decimal of a day.
(19) 26½ lbs. to the decimal of a ton.
(20) 15 cwt. 1 lb. to decimal of a ton.
(21) 5 oz. 12 dwts. 16 grns. to decimal of a pound.
(22) 2*s.* 8½*d.* to the decimal of 1*l.* 1*s.* 8*d.*
(23) 3 qrs. 3 lbs. 2 oz. 2·816 drams to the decimal of a ton.
(24) 9 lbs. Av. as a decimal of 5 lbs. Troy.
(25) 3⅛ of 15*s.* as the decimal of 10*s.* 6*d.*
(26) 7⅕ linear feet to the decimal of a rod.
(27) ⅜ of a rood to the decimal of ½ of an acre.
(28) 5 oz. 6 dwts. 18 grns. to the decimal of ¾ of *1 lb. Troy.*

(29) 5*s.* 6*d.* to the decimal of $\frac{4}{7}$ of a guinea.
(30) $\frac{3}{5}$ of $\frac{5}{8}$ of $\frac{4}{5}$ of 3*s.* 6*d.* to the decimal of 1*l.*

2. Find the value of

(1) ·625 of a shilling. (2) ·065625 of a £.
(3) 2·778125 of 6*s.* 8*d.* (4) 2·74 of 12*s.* 6*d.*
(5) 22·25 of 2*l.* 2*s.* 6*d.* (6) ·625 of a gallon.
(7) ·384 cwt. (8) ·325 of 1½ ton.
(9) ·0027 of a ton. (10) ·009943 of a mile.
(11) ·065 of an acre. (12) 13·62 of an oz. Troy.
(13) ·58 of a year. (14) ·5*s.* + ·7 crown + ·125*l.*
(15) ·175 of a ton + ·195 of a cwt. + ·145 of a qr.
(16) $\dot{0}0\dot{9}9$ of 1*l.* 5*s.* 3*d.*
(17) $2\cdot\dot{1}42857$ of a guinea.
(18) $\cdot4\dot{0}\dot{9}$ of a lb. Avoirdupois.
(19) $\cdot708\dot{3}$ of 1 lb. Troy.
(20) $5\cdot25l. + \cdot\dot{6}$ of a guinea + ·05 of 5*s.*
(21) ·6*l.* + ·3125 shilling + ·$\dot{2}$ guinea.

3. Add together ·8$\dot{3}$ of a shilling, ·0$\dot{3}$ of a halfcrown, and ·05416 of a pound; and also $\frac{1}{8}$ of a guinea, $\frac{3}{5}$ of a £, and 1·5 of 3*s.* 7*d.*

4. Subtract ·625 of a crown from 1·375*l.*, ·$\dot{3}$ of a crown from ·8$\dot{3}$ of 1*l.*, and ·854166 of 10*s.* from ·11$\dot{7}\dot{2}$ of 13*l.* 15*s.*

5. Multiply 227*l.* 9*s.* 11*d.* by 25·6.

6. Divide 3 qrs. 25 lbs. 15 dr. by 9·21, and 12 miles 4 furlongs 37 poles $8\frac{8}{33}$ yards by 4·05.

7. Reduce $\frac{1}{5}$ to a decimal; and taking the result as the decimal of a yard, find the value of it.

8. Reduce

(1) ·26*d.* to the decimal of 1*l.*
(2) 2·$\dot{9}$ halfcrowns to the fraction of 1*s.* 4*d.*
(3) $\frac{2}{3}$ of 1 guinea to the decimal of 1*l.* 3*s.* 4*d.*

9. Find the value of $\frac{3}{4}$ of $\cdot 12\dot{6}$ of 12*s.* 6*d.*, and reduce the result to the decimal of 5*l.*

10. The price of gold being 3*l.* 17*s.* $10\frac{1}{2}d.$ an ounce, find the weight of a nugget worth 341·0925*l.*

11. Find the value in English money of 1572·4185 francs when the exchange is at 26·675 francs per pound.

VIII. PRACTICE.

1. Find the cost of

(1) 3451 articles at 2*l.* 18*s.* 9*d.* each.
(2) 1005 articles at 3*l.* 18*s.* $11\frac{3}{4}d.$ each.
(3) 1175 articles at 5*l.* 16*s.* $9\frac{1}{4}d.$ each.
(4) 2354 articles at 1*l.* 5*s.* 6*d.* each.
(5) 1006 articles at 1*l.* 1*s.* $1\frac{1}{4}d.$ each.
(6) 735 articles at 16*s.* $10\frac{1}{2}d.$ each.
(7) 1961 articles at 17*s.* $11\frac{3}{4}d.$ each.
(8) 2791 articles at 15*s.* $5\frac{2}{3}d.$ each.
(9) $3115\frac{1}{4}$ articles at 15*s.* 6*d.* each.
(10) $213\frac{7}{8}$ articles at 5*l.* 2*s.* 6*d.* each.
(11) $187\frac{2}{3}$ articles at 2*l.* 3*s.* $6\frac{3}{4}d.$ each.
(12) $255\frac{15}{16}$ articles at 4*s.* 4*d.* each.

2. Find

(1) The value of 92 tons of hay at 6*l.* 18*s.* 9*d.* per ton.
(2) Total weight of 5792 iron bars, each weighing 23 lbs. 10 oz.
(3) Dividend on 1596*l.* at 15*s.* $9\frac{3}{4}d.$ in the £.
(4) Duty on 3 qrs. 12 lbs. at 5*s.* 3*d.* per lb.
(5) *Rent of* a field 880 yards long and 660 yards *wide at* 2*l.* 15*s.* per acre.

3. A bankrupt owes 3549*l.*, and can pay 17*s.* 6*d.* in the pound: what are his assets worth, and what loss do his creditors sustain?

4. Find the cost of

(1) 2 cwt. 2 qrs. 17 lbs. at 1*l.* 10*s.* 4*d.* per cwt.
(2) 42 cwt. 16 lbs. at 5*l.* 16*s.* 8*d.* per cwt.
(3) 23 cwt. 3 qrs. 12 lbs. at 1*l.* 7*s.* 6*d.* per cwt.
(4) 75 cwt. 3 qrs. 21 lbs. at 5*l.* 15*s.* 6*d.* per cwt.
(5) 13 lbs. 7 oz. 15 dwts. at 7*l.* 11*s.* 4*d.* per lb.
(6) 2 qrs. 23 lbs. 8 oz. at 2*l.* 6*s.* 8*d.* per cwt.
(7) 3 qrs. 1 nail at 16*s.* 8*d.* per yard.
(8) 15 dwts. 12 grns. at 4*l.* 4*s.* per oz.
(9) 7 weeks 5 days at 3 guineas per month of 4 weeks.
(10) 3 qrs. 13 lbs. 7 oz. at 7*l.* 11*s.* 4*d.* per lb.

5. Find

(1) The duty on 179 cwt. 2 qrs. 12 lbs. at 2*l.* 3*s.* 6½*d* per cwt.
(2) The rent of 956 acres 3 roods 10 perches of land at 28*s.* an acre.
(3) The rent of 215 acres 1 rood 5 perches at 2 guineas an acre.
(4) The rent of 23 acres 3 roods 5 poles at 2½ guineas per acre.

6. What must be given for a lump of gold weighing 2 lbs. 3 oz. 16 dwts. at 3*l.* 17*s.* 6*d.* per oz.?

7. If 1 cwt. cost 2*l.* 9*s.* 6*d.*, what will 45 cwt. 3 qrs. 14 lbs. cost?

8. What is the dividend on 1710*l.* 14*s.* 6*d.* at 13*s.* 4½*d.* in the £?

9. A bankrupt's liabilities are estimated at 3768*l.* 17*s.* 6*d.*: what are his assets if he can pay 13*s.* 7½*d.* in the £?

IX. MENSURATION.

1. Find the area of

(1) The floor of a room 16 feet 8 inches long and 15 feet 9 inches wide.

(2) A wall 20 feet 4 inches long and 6 feet 6 inches high.

(3) An enclosure $50\frac{3}{4}$ yards long and $20\frac{1}{2}$ yards wide.

(4) The whole surface of a room 22 feet 5 inches long, 18 feet 4 inches broad, and 10 feet 9 inches high.

2. If a board be 9 inches broad, what must be its length if it contain 15 square feet?

3. How many yards of carpet $\frac{1}{2}$ a yard wide will be required for a room 12 yards long by 6 yards 1 foot 6 inches broad? and how many for another room $22\frac{1}{2}$ feet by 19 feet?

4. What will be the cost of carpet $\frac{3}{4}$ yard wide at 3*s.* 9*d.* per yard for a room 18 feet 9 inches long and 12 feet 6 inches wide?

5. How many tiles, each 9 inches by 6 inches, will be required to pave a court 24 feet 6 inches long and 15 feet 9 inches wide?

6. Westminster Hall is 237 feet long by 68 wide; Brussels Town Hall 264 by 50: how many more persons could the former accommodate than the latter, allowing $\frac{2}{3}$ of a square yard to each person?

7. How many people can be seated in a room 108 feet long and 55 feet wide if $4\frac{1}{2}$ square feet are allowed for each person? And what must be the length of a room *66 feet wide* which would hold the same number of *people?*

8. Find the area of the walls of a room 27 feet long, 23 feet 6 inches wide, and 16 feet high.

9. How many yards of paper 2 feet 3 inches wide are required for the walls of a room 24 feet long, 18 feet 9 inches wide, and 14 feet 2 inches high?

10. If tin foil is sold in sheets 10 inches wide, what length of it will be required to line a box 16 inches wide, 15 inches high, and 20 inches long?

11. What will be the cost of painting the walls of a room 17 feet 3 inches long, 15 feet 5 inches wide, and 13 feet 6 inches high, allowing 54 square feet for windows, &c., not to be painted, at 9*d.* per square yard?

12. What will it cost to cover a platform 12 feet square and $2\frac{1}{2}$ feet high with cloth at 2*s.* 6*d.* the square yard?

13. How much paper 20 inches wide will be required for the walls of a room 19 feet 6 inches long, 17 feet 3 inches wide, and 10 feet high? And what will it cost at $2\frac{1}{4}$*d.* per yard?

14. A room is 29 feet long, 18 feet 3 inches wide, and 15 feet 6 inches high. Find

(1) The area of the floor.
(2) The area of the walls.
(3) The cost of a cornice 18 inches deep all round the room at 3*s.* 6*d.* the square yard.

15. If the ceiling of the room in Question 13 is painted at 2*s.* $1\frac{1}{2}$*d.* per square foot, and a cornice 5 inches deep is run round at 1*s.* 3*d.* per square foot, what is the cost of these decorations?

16. A room 20 feet 3 inches long by 15 feet 9 inches wide is painted at 3*d.* per square yard, and costs 27*s.*; what is the height of the room?

17. *The area of* the floor of a room 24 feet 3 inches

long is 42 square yards 1 foot 132 inches: what is the breadth? And if the cost of painting the four walls at 4*s.* per square foot be 183*l.* 12*s.* 4*d.*, find the height of the room.

18. A rectangular enclosure, 50 yards 2 feet 3 inches long, and 20 yards 1 foot 6 inches wide, has a path 4 feet 6 inches wide taken out of it all the way round: what is the area of the remaining space?

19. A path 6 feet wide is made across a field at one end and down one side. If the field is 79 yards long and 64 yards wide, what is the area of the path? And what part is it of the whole field?

20. How many cubic feet of air are there in a room which is 90 feet long, 40 feet wide, and $22\frac{3}{4}$ feet high? And by how much would the volume be increased by raising the ceiling 9 inches?

21. If the length of a cubit be 22 inches, what is the solid content of a room 120 cubits long, 18 high, and 45 broad?

22. A certain trench being 50 feet long, 4 feet wide, and $2\frac{1}{2}$ feet deep, what will be the depth of another trench which, being 45 feet long and 5 feet wide, contains 50 cubic feet less than the other?

23. What length of a plank 6 inches broad and 4 inches thick will be equivalent to a solid foot?

24. What is the depth of a cistern whose base contains 15 square feet, and which holds 2 cubic yards of water?

25. A hollow rectangular vessel, formed of material one inch thick, and whose external dimensions are 13 feet 6 inches, 6 feet 8 inches, and 6 feet 11 inches respectively, weighs 1 cwt. 1 qr. and 10 lbs. Find the weight *of a solid mass of* the same material and the same *external dimensions.*

X. SIMPLE PROPORTION.

1. If 14 cwt. 12 lbs. of sugar cost 46*l.* 1*s.* 8*d.*, what would be the cost of 18 cwt. 63 lbs.?

2. If a railway fare for $57\frac{1}{2}$ miles be 11*s.* 6*d.*, what ought to be the fare for $216\frac{1}{4}$ miles?

3. If a quantity of wheat fills 1155 sacks, each holding 8 bus. 0 pks. 1 gal., how many sacks will it take to contain it, each holding 6 bus. 0 pks. 0 gals. 3 qts.?

4. A watch loses 2 hrs. 13 min. a week: how much does it lose in 36 hours?

5. If a person travelling 12 hours a day perform a journey in 24 days, in what length of time will he perform the same journey if he travel 16 hours a day?

6. A railroad running inland for 56 miles has to reach a level 896 feet above its terminus on the coast: shew that the average inclination of the line is 1 foot in 330.

7. How much carpet at 7*s.* 6*d.* per yard must be given in exchange for 84 reams of paper at 1*l.* 12*s.* 6*d.* per ream?

8. How much wine at 2*l.* 10*s.* per dozen must be given in exchange for 3 puncheons of rum at 60*l.* 8*s.* 4*d.* per puncheon?

9. A person paid 17*l.* 6*s.* 6*d.* income tax at 7*d.* in the pound: what does he pay at 5*d.*?

10. What will be the gain to a person whose annual income is 1251*l.*, if the income tax is reduced from 16 pence to 7 pence in the £?

11. A merchant failing for 10000*l.* has goods and debts to the amount of 3750*l.*: how much can he pay in the pound?

12. If $2\frac{1}{4}$ lbs. of cotton cost 1*s.* $1\frac{1}{2}$*d.*, how much may be bought for 2*s.* 6*d.*?

13. If $\frac{5}{6}$ of a yard cost $\frac{2}{3}$ of 1*l.*, what will $\frac{11}{12}$ of a yard cost?

14. If $\frac{5}{7}$ of an estate be worth 1003*l.* 17*s.* 1*d.*, what is the value of $\frac{2}{5}$ of it?

15. If $4\frac{5}{9}$ oz. Av. cost $8\frac{3}{32}$*s.*, what will $8\frac{1}{2}\frac{3}{4}$ lbs. cost?

16. A and B start at a distance of 64 miles from each other; A walks $3\frac{1}{2}$ miles in the hour, B $2\frac{1}{2}$; if they start at the same time how many miles will each walk before they meet?

17. If 4 men or 6 women can do a piece of work in 20 days, how long will 3 men and 5 women take to do it?

18. A garrison of 2310 men is provisioned for half a year; at the end of six weeks they receive a reinforcement of 500 men: how long will their provisions now last, reckoning 365 days to the year?

19. If a man earns 7*l.* 10*s.* in 3 weeks, and lays by one-half of it, how much will he save in a year of 52 weeks?

20. From what property of Proportion is the Rule of Three derived? If a bankrupt has assets to the amount of 1020*l.*, and debts to the amount of 3225*l.*, what will his creditors receive in the £?

XI. COMPOUND PROPORTION.

1. If 27 men earn 35*l.* in 8 days, how much will 18 men earn in 11 days?

2. If 7 men will earn 10*l.* 7*s.* 6*d.* in $10\frac{1}{2}$ days, what sum will 28 men earn in $31\frac{1}{2}$ days?

3. *If it costs 86l. 8s.* to maintain 48 men for 16 days, *how much will it cost* to maintain 32 men for 12 days?

4. If 36 yards of cloth, 7 quarters wide, cost 25*l.* 4*s.*, what will be the cost of 120 yards of the same quality only 5 quarters wide?

5. If a carrier charges 1*s.* for the carriage of a parcel 10 lbs. weight 10 miles, how much should he charge for carrying a parcel 1 cwt. 8 lbs. weight 2 miles and a half?

6. If 20 men in 3 weeks earn 90*l.*, in what time will 12 men earn 150*l.*?

7. If 9 men reap a field of 8 acres in 12 hours, how many men will reap a field of 28 acres in 18 hours?

8. If 2 men can reap $2\frac{1}{2}$ acres in $2\frac{3}{4}$ days, how long will 11 men take to reap 15 acres?

9. If 6 men earn 8*l.* for 5 days' work, how long will it take 15 men to earn 4*l.*?

10. How many days of 15 hours each would 60 men take to do what 45 men can do in 30 days of 12 hours each?

11. If 10 men can reap $7\frac{1}{2}$ acres in 3 days of 12 hours each, how many days will it take 8 men to reap 9 acres, working 16 hours a day?

12. A courier travels 130 miles in 3 days of 12 hours each: in how many days of 10 hours each can he travel 360 miles at the same hourly rate?

13. A wall, that is to be built to the height of 27 feet, was raised 9 feet high by 12 men in 6 days: how many men must be employed to finish it in 4 days at the same rate of working?

14. If a tradesman with a capital of 770*l.* gain 60*l.* 10*s.* in $5\frac{1}{2}$ months, how long would it take him to gain 20*l.* with a capital of 100*l.*?

15. If a document takes 25 minutes to read for a man who reads 2 pages in 3 minutes, how long would a document of 3 times the length occupy a man who reads *a page a minute*?

16. There are two adjacent reservoirs. The one, which is 84 feet long, 27 feet wide, and 15 feet deep, is filled by a system of steam pumps in 9 hours. The same pumps take 48 hours to fill the other, which is 90 feet long by 42 feet wide. What is its depth?

XII. SIMPLE INTEREST.

1. Find the simple interest of

(1) 442*l.* at 4 per cent. per annum for 5 years.
(2) 541*l.* 13*s.* 4*d.* for $3\frac{1}{2}$ years at $5\frac{3}{4}$ per cent.
(3) 500*l.* 13*s.* 4*d.* for $2\frac{3}{4}$ years at $2\frac{3}{4}$ per cent.
(4) 10 guineas for 6 months at $3\frac{1}{2}$ per cent.
(5) 265*l.* 6*s.* 8*d.* at $2\frac{1}{2}$ per cent. for 3 years 73 days.
(6) 230*l.* for 125 days at 5 per cent.
(7) 125*l.* 16*s.* 8*d.* for 4 years 219 days at $3\frac{1}{2}$ per cent.
(8) 383*l.* 5*s.* from January 1 to February 19 at 5 per cent.

2. What will 3876*l.* produce at $4\frac{1}{2}$ per cent. in 7 years?

3. What will 1853*l.* 12*s.* 6*d.* amount to in 3 years at $2\frac{1}{4}$ per cent.?

4. Find the interest and amount of 535*l.* for 9 months at $4\frac{1}{2}$ per cent. per annum.

5. What sum of money will amount to

(1) 606*l.* 9*s.* 9*d.* in 2 years at 4 per cent.?
(2) 591*l.* 12*s.* 4*d.* in 4 years at $2\frac{1}{2}$ per cent.?
(3) 338*l.* 8*s.* in $2\frac{1}{2}$ years at $3\frac{1}{3}$ per cent.?

6. *A sum of money* has lain at simple interest for

5 years at 4 per cent.; it now amounts to 126*l.* 12*s.*: how much was it originally?

7. What sum will produce

(1) 56*l.* 14*s.* interest in $2\frac{1}{4}$ years at $4\frac{1}{2}$ per cent.?
(2) 230*l.* 12*s.* 6*d.* in $4\frac{1}{2}$ years at 5 per cent.?
(3) 477*l.* 15*s.* in 3 years at $3\frac{1}{2}$ per cent.?

8. What sum must be invested on March 9 to provide by October 14 an amount of 504*l.* 14*s.*, the rate of interest being 5 per cent.?

9. At what rate per cent. per annum will

(1) the simple interest of 375*l.* 10*s.* amount to 4*l.* 13*s.* $10\frac{1}{2}$*d.* in 3 months?
(2) 225*l.* produce 10*l.* 2*s.* 6*d.* in 2 years?
(3) 172*l.* 11*s.* 8*d.* be the principal whose amount at the end of 3 months is 174*l.* 14*s.* $9\frac{3}{4}$*d.*?
(4) 506*l.* 5*s.* be the interest upon 3125*l.* from May 26, 1872, to Dec. 31, 1873?

10. The interest of 408*l.* 0*s.* 5*d.* for 6 months is 8*l.* 3*s.* $2\frac{1}{2}$*d.*: find the rate per cent. per annum.

11. A sum of 320*l.* having been left unpaid for 2 years 73 days, 70*l.* 8*s.* interest is charged upon it: at what rate per cent. per annum is that?

12. What rate per cent. is equivalent to 2*d.* in the £?

13. In what time will

(1) 355*l.* 15*s.* produce 62*l.* 5*s.* 3*d.* interest at the rate of 7 per cent. per annum?
(2) 250*l.* amount to 272*l.* 10*s.* at 4 per cent.?
(3) 241*l.* 5*s.* be the interest upon 1825*l.* at 5 per cent.?

14. If 50*l.* principal gain 7*l.* 10*s.* interest in 9 months, *what principal will* gain 68*l.* in a year and a quarter?

15. If 5*l.* is the charge for a loan of 75*l.* for 10 months, what is the charge for a loan of 250*l.* for 6 months?

16. If 1375*l.*, put out for 3 years, produce 144*l.* 7*s.* 6*d.* interest, for how long must 1500*l.* be put out at the same rate to produce 105*l.*

XIII. COMPOUND INTEREST.

1. Find the compound interest of

(1) 500*l.* at 3 per cent. for 2 years.
(2) 750*l.* at 10 per cent. for 2 years.
(3) 20000*l.* for 4 years at $2\frac{1}{2}$ per cent.
(4) 300*l.* for 3 years at $2\frac{2}{3}$ per cent.
(5) 3333*l.* 6*s.* 8*d.* for 2 years at 3 per cent., and for $3\frac{1}{2}$ years at 5 per cent.

2. Find the amount at compound interest of 125*l.* at $3\frac{1}{2}$ per cent for 2 years.

3. Find the simple and compound interest on 516*l.* 13*s.* 4*d.* for $2\frac{1}{2}$ years at $3\frac{3}{4}$ per cent.

4. Find the difference between the simple and compound interest on 365*l.* 4*s.* $8\frac{1}{4}$*d.* for 3 years at 4 per cent.

5. What is the compound interest of 250*l.* for 2 years at 4 per cent. per annum, if the interest be payable half-yearly?

6. What principal will amount to 144*l.* 14*s.* $0\frac{3}{4}$*d.* in 3 years at 5 per cent. per annum compound interest?

7. In what time would the compound interest upon 3125*l.* amount to 322*l.* 12*s.* at 4 per cent.?

8. If 1250*l.* amount in 2 years to 1458*l.* at compound *interest, what is the* rate per cent.?

9. *Explain Simple* and Compound Interest.

XIV. PRESENT VALUE AND INTEREST.

1. Find the present value

(1) Of 123*l.* due six months hence, interest being reckoned at 5 per cent. per annum.

(2) Of a bill for 369*l.* 9*s.* 0*d.* due in 7 months at $4\frac{1}{2}$ per cent.

(3) Of 598*l.* 9*s.* 9*d.* due at the end of 1 year 115 days at $2\frac{1}{2}$ per cent.?

2. What is the discount

(1) On 275*l.* 11*s.* $2\frac{1}{4}$*d.* for 7 months at 5 per cent. per annum?

(2) On 252*l.* 19*s.* 3*d.*, due a year hence, at $3\frac{1}{4}$ per cent.?

(3) On 151*l.* 6*s.* 3*d.* for 3 months, at $3\frac{1}{4}$ per cent. per annum?

3. What is saved by paying a debt of 205*l.* 16*s.* 8*d.* 12 months before, instead of 6 months after it is due, interest and discount at 4 per cent.?

4. If a person accepts a present payment of 172*l.* 11*s.* 8*d.* instead of 174*l.* 14*s.* $9\frac{3}{4}$*d.* due 3 months hence, what rate of discount does he allow?

5. Find the discount upon the following bills:—

(1) For 5376*l.* drawn March 28 at 9 months, and discounted May 26 at 4 per cent.

(2) For 1789*l.* 4*s.* drawn April 9 at 4 months, and discounted May 31 at 4 per cent.

(3) For 509*l.* 15*s.* $1\frac{1}{4}$*d.* drawn November 10 at 6 months, and discounted March 1, at $4\frac{1}{2}$ per cent.

6. Explain the difference between interest and discount.

XV. STOCKS.

1. Find the income produced by 26260*l.*, if invested in the $3\frac{1}{2}$ per cents. at 91.

2. Lay out 11677*l.* 10*s.* in the $3\frac{1}{2}$ per cents. at $86\frac{1}{2}$: what will be the half-yearly dividend?

3. A invests 552*l.* in the $3\frac{1}{4}$ per cents. when they are at 92; B invests 679*l.* in the 3 per cents. when they are at 97: find the difference of their incomes.

4. What would be the difference of income produced by 1000*l.* invested in the 4 per cents. at 98, and in the $3\frac{1}{2}$ per cents. at 94?

5. A person invested 3963*l.* 15*s.* in the 3 per cents. at $94\frac{3}{8}$: what was his income from the investment after paying an income tax of 7*d.* in the £?

6. What income will be obtained by investing 3790*l.* 8*s.* 5*d.* in the 3 per cents. at $92\frac{1}{2}$, $\frac{1}{8}$ per cent. commission on the stock purchased being charged by the broker?

7. How much more income will a person obtain by investing 3345*l.* in 3 per cents. at $83\frac{1}{2}$ than in $3\frac{1}{2}$ per cents. at $111\frac{3}{8}$, brokerage in each case being $\frac{1}{8}$ per cent.?

8. What rate per cent. upon the money invested is obtained by purchasing

(1) 3 per cent. stock at 90?

(2) 5 per cents. at 105?

9. A and B had each a legacy of 5500*l.*; A invested it in 3 per cents. at $82\frac{1}{2}$, and B in the $3\frac{1}{2}$ per cents. at 96: which gained the higher rate of interest, and what was the difference between their incomes?

10. If I buy 3 per cents. at 90, and $3\frac{1}{2}$ per cents. at $95\frac{4}{11}$, which is the better investment? and by how much per cent.?

11. An exchequer bill costs 100*l.*, and bears interest at 2*d.* per day; 100*l.* three per cent. consols cost 96*l.*: which investment gives the best interest for money, and by how much per cent.?

12. How much 3 per cent. stock at 91 must be sold to realise 473*l.* 4*s.*? and what income will be sacrificed by parting with it?

13. What sum must be paid for 1340*l.* 12*s.* stock at 110¼ per cent.?

14. What sum must be given for 3785*l.* stock at 95⅜, with 2*s.* 6*d.* per cent. commission?

15. Find the income produced by 12600*l.* 3 per cent. stock, and its sterling value at 95.

16. What is the price of consols when 250*l.* stock costs 214*l.* 7*s.* 6*d.*?

17. How much money must be invested in the 3 per cents. at 92 to secure an annual income of 150*l.*?

18. How much 3¼ per cent. stock at 97½, and how much 3 per cent. stock at 90, must be purchased to yield an income of 150*l.* in each case? Show that both will cost the same if there is no brokerage charged. What will each cost if brokerage is ⅛ per cent.?

19. At what price must I buy into the 3 per cents. to receive at the rate of 3¼ per cent.?

20. What do I gain or lose in income by selling 7800*l.* 3 per cents. at 90, and investing the proceeds in the 3½ per cents. at 104?

21. What would be the difference of income produced by investing in the 3 per cent. stocks at 85½ the sum of 1060*l.* previously lent on a mortgage at 4½ per cent.?

22. How much stock can be purchased by the transfer of 1000*l.* stock from the 3 per cents at 72 to the 4 per cents. at 90? and what difference in annual income will be produced by it?

23. Find the alteration in income arising from the transfer of

(1) 4550*l.* stock from the 3 per cents. at 89 to $3\frac{1}{4}$ per cents. at 91.

(2) 1365*l.* stock from the 3 per cents. at 90 to $4\frac{1}{2}$ per cents. at 105.

(3) 2775*l.* stock from the $3\frac{1}{2}$ per cents. at 99 to $3\frac{1}{4}$ per cents. at $92\frac{3}{8}$, $\frac{1}{8}$ per cent. commission being paid on the purchase.

24. How much 3 per cent. stock at 90 must be sold to realise 3735*l.*? and if this sum be invested at 5 per cent., what increase of income will it produce?

25. A person invests 1950*l.* in 3 per cent. stock at $93\frac{1}{8}$, paying $\frac{1}{8}$ per cent. brokerage; he receives one half-year's dividend, and then sells at 92, again paying brokerage: what has he made by the investment?

26. A person holding 2450*l.* 3 per cent. consols, sells out at $93\frac{1}{2}$, and invests the proceeds in Russian $4\frac{1}{2}$ per cents., whereby his income is increased by 31*l.* 13*s.* 9*d.*; at what price does he buy the Russian stock?

27. Egyptian 7 per cents. being at 40, in how many years' time might the debt be repudiated without loss to an investor reclaiming only simple interest, and considering the interest on safe investments to be 4 per cent.?

XVI. PROFIT AND LOSS.

1. A wine merchant mixes 5 gallons of wine at 32*s.* per doz., 4 at 45*s.*, and 1 at 55*s.*; how much will the *mixture be worth* per doz. (supposing the dozen to be *equal to 2 gallons*)?

2. A grocer has 72 lbs. of tea at 5*s.* 6*d.*, and 60 lbs. at 3*s.* 8*d.*; at what price can he sell them mixed?

3. If a grocer mixes 120 lbs. of sugar at 9*d.* a pound with 250 lbs. of sugar at 5½*d.* a pound, at what rate must he sell the mixture so as to gain a penny a pound?

4. If goods bought at 5*l.* 6*s.* 8*d.* per cwt. are sold at 111*l.* 9*s.* 4*d.* per ton, what is the gain per cent.?

5. At what price per cwt. must goods be sold which were bought at 1*l.* 5*s.* per ton, in order to gain 5 per cent.?

6. By selling cloth at 4*s.* 6½*d.* per yard 9 per cent. is gained: at what price per ell must the same be sold to gain 6½ per cent.?

7. If 12 per cent is gained by selling a cheese for 4*s.* 8*d.*, how much per cent. will be gained if it is sold for 5*s.* 9*d.*? and what will the actual gain be?

8. A man buys 6 score of apples at the rate of 3 for 2*d.*, and 5 score at ½*d.* each: if he sells them all at the rate of 4 for 3*d.*, how much and at what rate per cent. will he gain?

9. If 3 lbs. of tea at 6*s.* 6*d.* per lb., 5 lbs. at 5*s.*, and 7 lbs. at 2*s.* 6*d.* are mixed, at what price per lb. must the mixture be sold to gain 2½ per cent.?

10. Bought 276 yards of cloth at 4*s.* 8*d.* per yard; sold ⅓ of it at 6*s.*, ½ at 5*s.* 4*d.*, and the remainder at 4*s.* 2*d.*: what is the whole gain or loss, and the gain or loss per cent.?

11. If the manufacturer makes a profit of 20 per cent., the wholesale dealer a profit of 25 per cent. and the shopkeeper a profit of 40 per cent., what was the cost of the manufacture of an article bought at a shop for 17*s.* 6*d.*?

XVII. PROPORTIONAL PARTS.

1. Divide 81 into two parts which shall be in the ratio of 4 : 5, and also into parts in the ratio of $\frac{1}{4}$, $\frac{1}{5}$.

2. Divide 154*l.* between 4 persons, in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$; and 144*l.* between 3 persons in the proportion of $\frac{1}{5}$, $\frac{2}{5}$, $\frac{1}{2\,5}$.

3. A gentleman left by will to A 1000*l.*, to B 950*l.*, to C 800*l.*, to D. 750*l.*; his estate produced only 2800*l.*: how much will each legatee have to receive?

4. Three persons, A, B, C, freighted a ship with 340 tuns of wine, of which A loaded 110 tuns, B 97, C the rest; in a storm the seamen threw overboard 85 tuns: how much must each person sustain of the loss?

5. Divide 600*l.* between A, B, and C, so that B shall have twice as much as A, and C twice as much as A and B together.

6. Divide 100*l.* among A, B, C, and D, so that B may receive as much as A, C as much as A and B, and D as much as A, B, and C together.

7. Divide 202*l.* 7*s.* among 4 men, 3 women, and 5 children, so that each man may have twice as much as each woman, and each woman three times as much as each child.

8. A person divides 3*l.* 15*s.* amongst 25 men, 25 women, and 25 children; every man has as much as a woman and child put together, and every woman has twice as much as a child: how much has each?

9. A legacy of 2420*l.* is left to a family of three children in such proportions that after payment of a legacy duty of 15*l.* per cent. the eldest will receive twice as much as the second, and the second three times as *much as the youngest*: what will be their respective *shares?*

10. If 2 women and 4 boys can hoe a field 1 a. 2 r. 20 p. in a day, how long will it take 2 men and 2 boys to hoe a field of 6 acres and a half? the rates of working of a man and woman being as 3 to 2 and of a woman and boy as 2 to 1.

11. A combined with 3 other persons to import a chest of tea. These 3 were to take equal portions, but A agreed to take a double portion. This he disposed of again at cost price among 4 other persons equally, reserving as before a double portion for himself. What portion of the whole cost will fall upon A?

12. If a man earns twice as much as a woman and four times as much as a boy, when 7 men, 9 women, and 8 boys earn between them *4l.* 1*s.* per diem, how much will 4 men, 3 women, and 5 boys earn in a week?

13. If 64 gals. of wine and 16 gals. of water are mixed, how much wine is there in $2\frac{1}{2}$ pints of the mixture?

14. If 2 lbs. 6 oz. of copper are mixed with 1 lb. 10 oz. of tin, what weight of each metal is there in a coin weighing $4\frac{1}{2}$ oz.?

15. Two parcels of gunpowder contain nitre, sulphur, and charcoal in the proportion of ·85, ·10, ·05 and ·80, ·12, ·08 respectively: how much of each ingredient will there be in 2 lbs. of gunpowder made by taking 1 lb. out of each parcel?

16. A, B, C rent land; A puts 500 sheep on it for 4 months; B, 600 for 3 months; C, 1200 for 2 months: if the whole rent amounts to 116*l.* 5*s.*, how much has each to pay?

17. A, B, C contribute equal sums towards an enterprise; at the end of 6 months A buys $\frac{1}{2}$ of B's share and $\frac{1}{3}$ of C's: what share of the profits at the year's end will each be entitled to?

18. Four men working 15 hours, 3 men 12 hours, and

8 men 3 hours, jointly accomplish a work for which they are paid 6*l.* 10*s.*: find the share of each man in each set, and also a man's wages per week of 6 days and 6 hours a day.

19. If a man and a boy earn 25*s.* in 5 days, and 2 men and 3 boys earn 48*s.* in 4 days, how much would 3 men and 5 boys earn in a week?

XVIII. ROOTS AND POWERS.

1. Find the square root of

(1) 61009. (2) 10609. (3) 480249.
(4) 942841. (5) 190096. (6) 4826809.
(7) 10310521. (8) 13741849. (9) 66585600.
(10) 1624·09. (11) 187·4161. (12) 40·1956.
(13) 1524·9025. (14) $\frac{289}{1369}$. (15) $1\frac{21}{100}$.
(16) $\frac{729}{12225}$. (17) $\frac{7056}{9216}$. (18) $1\cdot\dot{7}$.
(19) $7\cdot\dot{1}$. (20) $17\cdot36\dot{1}$.

2. Find correct to six figures the square root of

(1) 7875986. (2) 908·209. (3) ·4325.
(4) 235·6. (5) 2·1. (6) 152·49025.
(7) ·0007612. (8) 442·707849. (9) $40\frac{1}{36}$.

3. Find to three places of decimals the square roots of

(1) 2. (2) 3. (3) 19. (4) 20.
(5) 101. (6) $\frac{3}{5}$. (7) $\frac{3}{2}$. (8) $2\frac{1}{6}$.

4. Extract the square roots, and where not exact to three places of decimals, of

(1) ·25, 2·5, ·025, ·00025.
(2) 16, ·016, ·16, ·0016.
(3) 729, 72·9, ·0729, ·0000729.

5. The area of the floor of a square room is 35 square yards 0 ft. 9 in.: what is the length of each side?

6. A square fish-pond contains an acre; find the length of each side.

7. The surface of a square table contains 1387·5625 square inches; what is the length of a side?

8. Extract the cube root of

(1) 226981.	(2) 389017.
(3) 373·248.	(4) 2863288.
(5) 12·977875.	(6) 259694072.
(7) ·125.	(8) 1·157625.
(9) ·000125.	(10) ·001157625.

9. Find the cube of ·094, of ·0$\dot{3}$, of $\frac{3}{5}$, and of $2\frac{2}{3}$ of $\frac{15}{16}$.

10. Find the 4th power of ·8, of 9, of ·09, and of $\dot{9}$.

11. Account for the use and position of dots set over certain figures in evolution, and shew that the number of decimal places in every square decimal is even.

XIX. MISCELLANEOUS.

1. If 15 cows are worth 37 sheep, find the value of a cow when 3 sheep are worth *22l.* 10*s.*

2. If 1 lb. of tea is worth 50 oranges, and 70 oranges are worth 84 lemons, what does tea cost per lb. if a lemon costs a penny?

3. If A can walk 9 miles while B walks 14, and B can walk 7 miles while C walks 6, what start should C give A in a 10-mile match?

4. When a pound sterling is worth on the exchange 25·05 francs, a man sells 45 pieces of 20 francs at 16*s.* 2*d.* *each; how much* does he gain?

5. The centigrade thermometer in which water freezes at zero, and boils at 100°, stands at 25°; what will Fahrenheit's thermometer mark in which water freezes at 32° and boils at 212°?

6. A pays 21*l.* 4*s.* income tax at the rate of 5 per cent. on his income; B pays 18*l.* 5*s.* at the rate of 3 per cent.: what is the difference between their incomes?

7. A farmer makes 3 cheeses in 2 days during one-half the year, and 2 cheeses in 3 days during the other half: how many does he make in 12 days on the average?

8. A person with an income of 500*l.*, on which he has to pay 2½*d.* in the pound every quarter as income-tax, finds that the payments have amounted to 26*l.* 0*s.* 10*d.* in 12 months: how many quarters are there in a year?

9. The gross receipts of a railway company are 46220*l.*: after deducting 35 per cent. for working expenses, and paying a preference dividend of 6 per cent. on a capital of 500000*l.*, what sum is available for division among the shareholders?

10. When the rates of exchange between England and France and England and America are 5*s.* = 6 francs, and 1 dollar = 4s. 10*d.*, how many francs will be = 290 dollars?

11. If the fare from London to Marseilles be 6*l.* 5*s.* 3*d.*, from Marseilles to Civita Vecchia 133 francs, and from Civita Vecchia to Rome 1 scudo 86 baiocchi, how many francs would a ticket to Rome cost, reckoning 1*l.* = 25 francs and the scudo at 100 baiocchi of one halfpenny each?

EXAMINATION PAPERS.

ARITHMETICAL.

I.

1. A bankrupt's debts amount to 3625*l.* 16*s.* 3*d.*, and his assets to 1208*l.* 12*s.* 1*d.*: how much can he pay in the £?

2. Reduce $\frac{5}{6}$ of a halfcrown to the fraction of a shilling, and $\frac{2}{7}$ of a cwt. to the fraction of a lb.

3. What will it cost to carpet a room $18\frac{3}{4}$ feet long and $17\frac{1}{2}$ feet wide with carpet $3\frac{1}{2}$ feet wide at $\frac{1}{3}$*l.* per yard?

4. Add together 29·0146, 3146·5, 2109, ·62417, 14·16.

5. Divide ·101 by ·5, ·05, 50, and 5000.

6. Reduce to vulgar fractions ·01034 and 9·9.

7. Add together $\frac{2}{3}$ of $1\frac{1}{2}$, $\frac{11}{12}$ of $\frac{9}{15}$, $\frac{7}{8}$ of $3\frac{2}{5}$.

8. Divide $\frac{9}{11}$ of $\frac{19}{81}$ by $\frac{33}{153}$ of $\frac{114}{121}$.

9. How many yards of canvas 18 inches wide would it take to cover a box 4 ft. long, 3 ft. high, and three-quarters of a yard wide?

10. If 1025*l.* are put out to simple interest for 13 years at $4\frac{1}{2}$ per cent., what will be the interest and amount?

11. Find the square root of 488601 and of 84·82410.

12. If 7 men can mow 6 acres in 12 hours, how many men will mow 15 acres in 14 hours?

13. If $\frac{3}{8}$ of a yard of velvet cost $\frac{2}{5}l.$, what will $\frac{5}{16}$ of a yard cost?

14. The quick time, or step, in marching being two paces per second, or 120 per minute, at 28 inches each, at what rate per hour does a company march on a good road? and how long will they be in arriving at a garrison 20 miles distant, allowing a halt of 1 hour by the way for refreshment?

II.

1. Add together $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{9}$, $\frac{11}{12}$.

2. Find the G.C.M. of 2526 and 2947.

3. Subtract ·002 from 11, and multiply ·01 by 1·01.

4. Divide $2\frac{6}{7}$ of $1\frac{2}{5}$ by $3\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{1}{4}$.

5. If ·5 of 1*l.* buys ·4 of a gallon, how much may be bought for 6*s.* 3*d.*?

6. Divide 1·44 by 1·2, by 12, by ·012, and by 120.

7. Find the simple interest on 375*l.* at 4 per cent. for 11 years.

8. Divide 1300*l.* into three parts in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

9. Find the square root of 449·44 and of ·0001.

10. What is the cost of carpeting a room 21 ft. by 30 ft. with carpet $\frac{1}{2}$ a yard wide at 4*s.* 6*d.* a yard?

11. If 12 men do a piece of work in 15 days of 10 hours each, how many days of 9 hours will 10 men take to do it?

12. Find the income produced by investing 4500*l.* in the 3 per cents. at 90, and that produced by investing 2850*l.* in the $3\frac{1}{2}$ per cents. at 95.

13. A steam engine and a horse start together; the engine does the first mile in 5 minutes, the horse in 4; after this mile the pace of the engine is 30 miles per hour, that of the horse 20: which wins in a 3-mile race, and by how much?

III.

1. Divide

(1) 1·69 by ·13;
(2) 12·1 „ ·011;
(3) 001 „ ·01;

2. Reduce 12*s.* 6*d.* to the decimal of 1*l.*, and 7 lb. to the decimal of 1 cwt.

3. Divide $\frac{1}{4}$ of $\frac{1}{5}$ of $4\frac{2}{7}$ of $2\frac{1}{3}$ by $\frac{1}{9}$ of $4\frac{1}{5}$ of $3\frac{3}{4}$ of $1\frac{1}{7}$.

4. Find the compound interest on 100*l.* for 2 years at 10 per cent.

5. If 24 men do a piece of work in 14 days of 9 hours, how many will do it in 12 days of 7 hours?

6. Extract the square root of 17161.

7. Find the L.C.M. of 10, 14, 15, 21, 30, 42.

8. What sum invested in the 3 per cents. at 92 will produce an income of 100*l.* per annum?

9. If cotton is bought at 2*l.* 6*s.* 8*d.* per cwt., at what price per lb. must it be sold to bring in 5 per cent. profit?

10. What will it cost to carpet a room 20 ft. 3 in. long by 16 ft. wide with carpet 2 ft. 3 in. wide at 3*s.* 4*d.* per yard?

11. A and B together do a piece of work in 5 days; A alone would do it in 7 days: in how many days would B alone do it?

IV.

1. Find the value of an acre of land at $8\frac{1}{2}$*d.* per square foot.

2. Add together $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{11}{12}$, and express the sum in decimals.

3. Simplify

$$(1)\ \frac{1}{2+\dfrac{3}{4+\frac{5}{6}}}, \qquad (2)\ \frac{\frac{2}{9}\times\frac{3}{8}\div 2^{\cdot}}{\frac{1}{6}\{\frac{4}{5}-\frac{1}{16}\}}$$

4. Reduce $\frac{4}{5}$ of 3*s.* 4*d.* to the fraction of a guinea, and 6·72 oz. to the decimal of a cwt.

5. Divide 36·33 by 210, 35 by ·05, ·002 by 200.

6. Find the square root of 2209 and of 22·1841.

7. Find the simple interest on 2066*l.* 15*s.* 10*d.* for 4 years at $2\frac{1}{2}$ per cent., and the compound interest on 9030*l.* for 2 years at $3\frac{1}{3}$ per cent.

8. In how many days of 12 hours each would 72 men do a piece of work which 60 men can do in 30 days of 9 hours each?

9. Find the difference between the discount allowed at 5 per cent. on 810*l.* paid 3 months before it is due, and the interest charged for the loan of the same sum for the same period at the same rate.

10. A grocer mixes 13 lbs of tea at 3*s.* 4*d.* with 3 lbs. at 4*s.* 10*d.*: at what price must he sell the mixture so as to gain a penny a pound?

11. Find the value of $3\frac{2}{5}l. + 7\frac{2}{3}s. + 11\frac{3}{7}$ of a guinea, and of ·35 of a ton + ·39 of a cwt. + ·44 of a qr.

V.

1. Find the G.C.M. of 124321 and 197451.

2. Reduce to its lowest terms

$$\left\{1+\frac{1}{3+\dfrac{1}{3+\frac{1}{3}}}\right\} \div \left(\tfrac{1}{3}+\tfrac{1}{11}\text{ of }10\tfrac{2}{3}\right).$$

3. Find the value of $\frac{1}{480}$ of £ + $\frac{2}{15}$ of half a crown + $\frac{5}{42}$ of a guinea + $\frac{15}{2}$ of a penny.

4. Give the rule (1) for placing the point in division of decimals, (2) for reducing a circulating decimal to a vulgar fraction.

5. Reduce ·075 of a £ to the decimal of 4*s.* 6*d.*; and 2 square poles 9 yards 4 feet 72 inches to the decimal of 1 acre 60 square yards.

6. Find by Practice the value of 2 tons 7 cwt. 56 lbs. at 2*l.* 10*s.* 6*d.* per cwt.

7. A man buys eggs at 11$\frac{1}{4}$*d.* per dozen, and sells them at 2*d.* each: what does he gain per cent.?

8. Nine men were engaged to dig a trench 3 feet deep, 4$\frac{1}{2}$ feet wide, and 120 yards long in a certain time; but before it was begun it was discovered that the trench must be 5 feet deep, 180 yards long, but only 4 feet wide: how many men must be employed in order that it may be completed in the required time?

9. Find the simple interest on 387*l.* 10*s.* for 5$\frac{1}{4}$ years at 3$\frac{1}{2}$ per cent.

10. What sum, if laid out at 3$\frac{1}{2}$ per cent. for 6 years, will produce 28*l.* 17*s.* 6*d.* interest at simple interest?

11. What will be the cost of papering a room 14 feet 6 inches long, 10 feet 6 inches broad, and 8 feet high with paper 2 feet wide at 7$\frac{1}{2}$*d.* per yard?

VI.

1. Find the L.C.M. of 3, 5, 7, 9, 15, 63.

2. Divide $1 - (\frac{1}{2} + \frac{1}{3} + \frac{1}{24})$ by $1 - \frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{24}$, and find the value of $\frac{\frac{1}{6} + \frac{1}{8}}{8\frac{1}{2} + \frac{3}{14}}$.

3. Reduce 1 lb. Troy to the fraction of 1 lb. Avoirdupois.

4. Multiply ·000725 by 31·25, and divide the product successively by 6·25, 625, and ·0625.

5. Reduce to vulgar fractions ·0015625, ·05, ·03$\dot{1}\dot{8}$.

6. Find the value of 3·15625 of a £.

7. Find the square root of 106929 and 4·1209.

8. At the census of 1851 the population of Oxford was 27843; at the census of 1861 the population was 27560: what was the decrease per cent.?

9. A can walk 5 miles while B is walking 4: supposing A to walk 6 hours a day, and B 7 hours a day, how many days will B take in walking a distance which A can accomplish in 14 days?

10. Find the simple interest on 354*l.* 3*s.* 4*d.* for $3\frac{1}{3}$ years at $2\frac{1}{4}$ per cent.

11. A quadrangle is 50 feet long by 40 feet broad; it is crossed in each direction by a path 10 feet broad, and the remainder has to be turfed: how many strips of turf $1\frac{1}{2}$ foot long and 6 inches broad will be required for the purpose?

VII.

1. Find the value of

$$\frac{\frac{7}{10} - \frac{2}{3}}{\frac{9}{12} + \frac{1}{6}} \div \frac{\frac{1}{3}}{9\frac{1}{2}} \text{ and of } \frac{1}{2} + \frac{2}{9} - \frac{4}{15} + \frac{5}{18}.$$

2. 4500*l.* is divided between A, B, C, D; A receives $\frac{1}{2}$; B, C, each $\frac{2}{5}$ of the remainder: how much is left for D?

3. Reduce 1 lb. 12 oz. (Avoirdupois weight) to the *fraction of 1 cwt. 2* qrs., and $\frac{5}{16}$ of 6*s.* 8*d.* to the fraction *of a guinea.*

4. Express as vulgar fractions ·01375, ·0$\dot{3}$, 7·193$\dot{1}\dot{8}$, and divide ·175 by ·25, by 2·5, and by ·0025.

5. Find the value of ·3 of a guinea + ·125 of a pound + ·208$\dot{3}$ of a shilling + ·5 of a penny; and bring 10 weeks 3 days to the decimal of a year.

6. A can walk 10 miles in $2\frac{1}{4}$ hours; B can walk 11 miles in $2\frac{1}{2}$ hours; they start to walk a match from London to Oxford, a distance of 55 miles; which will arrive first, and by what amount of time will he win?

7. The profit made by selling beer at 1*d.* per pint above cost price is 50 per cent.: what is the gain per cent. made by selling it at 6*d.* a gallon above cost price?

8. Find the compound interest on 66*l.* 13*s.* 4*d.* for 2 years at 9 per cent.

9. What will be the cost of papering a room 20 feet long, 12 feet broad, 10 feet high, with paper $2\frac{2}{3}$ feet broad at $4\frac{1}{2}$*d.* per yard, an extra $\frac{1}{2}$*d.* per yard being charged for hanging?

10. A sovereign weighs $\frac{160}{123}$ of an oz. Troy: how many sovereigns may be coined out of a piece of metal weighing 1 lb. Troy? This metal being composed of $\frac{11}{12}$ pure gold and $\frac{1}{12}$ alloy, what is the value (expressed in money) of an oz. of pure gold?

VIII.

1. Find the G.C.M. of 5522 and 6006.

2. Multiply 2 acres 3 roods 25 poles by 16.

3. Divide $\frac{1}{12} - \frac{1}{18}$ by $3\frac{1}{75} - 2\frac{24}{25}$.

4. Find the value of $\frac{2}{5}$ of 1*l.* + $\frac{1}{3}$ of 4*s.* + $\frac{1}{10}$ of 6*s.* 8*d.*, and bring 16 poles to the fraction of a mile.

5. Bring to vulgar fractions ·06125 and ·12$\dot{3}\dot{7}$.

6. Reduce 2 oz. 13 dwts. to the decimal of 1 lb., and $\frac{2}{3}$ *of 13s. 6d. to* the decimal of 1*l.*

7. If a room is 27 ft. 5 in. long, 14 ft. 7 in. wide, 12 ft. 10 in. high, how much paper $\frac{7}{8}$ of a yard wide is required to cover the walls?

8. A piece of work can be done in 50 days by 35 men working together: if after 12 days 16 men strike, how soon will the remainder finish the work?

9. What is the amount at simple interest of 1275*l.* in $5\frac{2}{3}$ years at $3\frac{1}{3}$ per cent.?

10. Find the amount in 3 years of 180*l.* at $4\frac{1}{2}$ per cent. compound interest.

11. A pit 2 feet by 7, and 24 feet deep, cost 3*l.* to dig out: how deep will a pit be which is 7 feet by 9, and costs 4*l.* 10*s.*?

IX.

1. Add together 31 guineas, 199 sovereigns, and 311 halfcrowns; and subtract the sum from 517 half-guineas.

2. Reduce $\frac{1365}{8985}$ and $\frac{44}{147\frac{1}{8}}$ to their lowest terms, and find the value of $24 \times \frac{8}{9}$ of $7 + \frac{5}{6}$ of $3 \times \frac{4}{15}$.

3. Find the L.C.M. of 12, 84, 120, 132, 156.

4. Multiply $\frac{\frac{3}{7} \text{ of } \frac{5}{6}}{\frac{1}{4} \text{ of } \frac{9}{11}}$ by $\frac{\frac{5}{8} \text{ of } \frac{9}{111}}{\frac{3}{7} \text{ of } \frac{5}{6}}$.

5. Express $\frac{61}{80}$, $\frac{473}{659}$, and $\frac{11}{12}$ of $\frac{1}{8}$ of $2\frac{26}{77}$ as decimals.

6. Reduce 4*s.* $2\frac{1}{2}$*d.* to the decimal of 5*s.*, and find the value of ·0375 of 1*l.*

7. What is the present worth of 717*l.* 7*s.* 1*d.* due in 7 months at $4\frac{1}{4}$ per cent. per annum?

8. Find the square root of 2900209 and 6577·21.

9. If I buy sugar at 1*l.* 12*s.* 8*d.* per cwt., at what rate per lb. must I retail it to gain $7\frac{1}{7}$ per cent.?

10. *If 8 men can reap* 40 acres of wheat in 7 days, how *many acres would* 24 men reap in 28 days?

X.

1. What is the sum, difference, product, and quotient of $3\frac{1}{8}$ and $2\frac{1}{9}$?

2. Find the G.C.M. of 210, 258, and 903, and the L.C.M. of the first nine numbers.

3. Reduce $\cdot 02916\dot{6}$ to a vulgar fraction, and $\frac{6}{55}$ to a decimal.

4. Simplify $3 + \frac{1}{7 + \frac{1}{16}}$ as a fraction, and find its value to three places of decimals.

5. Find the value of 1·1625 of 5*s.* and 3·2 of 6*l.*

6. How much air is contained in a room 10 ft. long, 8 ft. 6 in. broad, and 6 ft. 6 in. high?

7. Find the square root of $32\frac{1}{9}$, and of 300 to five figures.

8. What is the compound interest on 95*l.* 16*s.* 8*d.* for 2 years at 3 per cent.?

9. A cistern can be filled by one pipe in 20 minutes, and by another in 30 minutes: in how many minutes would both together fill it?

10. Indian $4\frac{1}{4}$ per cent. stock may be purchased at 91 what interest per cent. will this investment yield?

11. If a man can travel 360 miles in 12 days of 8 hours each, how many hours a day must he travel at the same rate to complete 450 miles in 20 days?

XI.

1. A bankrupt's debts amount to 309*l.*, and his assets to *193l. 2s. 6d.*: how much will he pay in the pound?

2. If 2 men earn 15*s.* in 3 days, how much can 7 men earn in 4 days?

3. Simplify

(1) $\dfrac{1}{3+\dfrac{3}{4\frac{1}{2}}}$. (2) $\dfrac{2\frac{1}{4}-1\frac{3}{5}}{8\frac{1}{2}+1\frac{9}{10}}$.

4. Find by Practice the cost of 8 cwt. 2 qrs. 16 lbs. at 2*l.* 5*s.* 6*d.* per cwt.

5. Express as decimals $\frac{1}{25}$, $\frac{3}{1\,0\,2}$, and reduce 15*s.* $9\frac{3}{4}$*d.* to the decimal of 1*l.*

6. What is the value of ·625 of a shilling, $\frac{1}{8}$ of 1*l.*?

7. Divide 298·89 by ·1107, and ·0482796 by 1·2.

8. Find the square root of 106929 and of ·000271.

9. What is the present worth of 75*l.* due 15 months hence at 5 per cent.?

10. I buy 1000*l.* of L. C. and D. R. stock at par, and I sell at $17\frac{1}{2}$: what do I lose by the transaction?

11. How many bricks 9 inches long and 4 inches broad will floor a square room that is 20 feet on each side?

XII.

1. A man owing his creditors 1207*l.* 15*s.* makes a composition with them at 3*s.* 6*d.* in the pound: what amount does he pay them?

2. If 12 men plough 48 acres of land in 8 days, in what length of time will 5 men plough $2\frac{1}{2}$ acres?

3. Find the L.C.M. of 3, 8, 15, 21, 18.

4. Simplify $\dfrac{3}{\dfrac{2\frac{1}{4}}{1+3\frac{1}{2}}-\frac{3}{16}\text{ of }1\frac{1}{3}}$ and $\dfrac{2\frac{3}{4}+3\frac{1}{6}}{7\frac{1}{8}-1\frac{7}{44}}$.

5. Express as decimals $\frac{1}{36}$, $\frac{7}{256}$; and as a vulgar fraction $\cdot\dot{1}23\dot{6}$.

6. Find the value of ·01375 of 1*l.*, and of $3\frac{11}{12}$ of half a crown.

7. Divide 22·4615 by ·167, and ·004095 by ·273; and multiply 163·2 by ·0731.

8. Find the square root of 700569 and of ·000196.

9. If I sell 2500*l.* 3 per cent. consols at 95, and invest the proceeds in debentures at par, paying $4\frac{1}{2}$ per cent., what difference does it make in my income?

10. What is the discount for nine months on 4200*l.* at 5 per cent. per annum?

11. What is the cost of papering a room 16 feet long, 12 feet wide, and $10\frac{1}{2}$ feet high with paper 21 inches wide at $3\frac{1}{2}d.$ per yard?

XIII.

1. A has $\frac{1}{3}$ of a trading vessel, B $\frac{1}{4}$, and C $\frac{1}{8}$, and the rest belongs to the master: what is his share?

2. A country gentleman drinks a pint of port every day during the last 40 years of his life: what is the whole quantity drunk, and its value at 2*s.* per pint?

3. Find the G.C.M. of 75582 and 42237.

4. If 2 acres will maintain 3 horses for 4 days, how long will 6 acres maintain 6 horses?

5. Find by Practice the cost of 3 cwt. 3 qrs. $17\frac{3}{4}$ lbs. at 14*s.* $9\frac{1}{2}d.$ per lb.

6. Simplify $\frac{2\frac{1}{6}+7\frac{1}{3}}{3\frac{1}{6}\times 9\frac{3}{7}}$ and give the sum of $\frac{5}{8}$, $\frac{3}{5}$ of $\frac{4}{7}$, and $\frac{8}{9}$ of 3.

7. What is the value of ·1275 of 1*l.*, and of $\frac{3}{4\frac{1}{2}}$ of 1 cwt.?

8. Divide 9·3828 by 1117, by 11·17, and by ·01117.

9. What is the difference between the simple and compound interest on 550*l.* for 3 years at 5 per cent.?

10. A merchant sold goods for 753*l.* more than he gave for them, and cleared 15 per cent.: what sum did he buy them for?

11. Two boats start from Iffley to row a race: the one rows 31 strokes a minute, and clears 30 feet every stroke; the other rows 33 strokes and clears 28 feet: in how long will the quicker boat of the two row the course (1 mile 720 yards), and how far will it be ahead when it passes the winning post?

XIV.

1. What is the price of a Turkey carpet 20 feet 6 inches long by 12 feet 9 inches wide at a guinea the square yard?

2. If I can write 300 pages of a book in 2 months working $7\frac{1}{2}$ hours a day, in how many months could I write 450 pages working 9 hours a day?

3. Simplify

$$\frac{\frac{1}{7} \text{ of } \frac{3}{4}}{1 - \frac{1}{1 + \frac{3}{25}}} \text{ and } \frac{3\frac{3}{4} \times 2\frac{1}{8}}{1\frac{1}{16} - 1\frac{1}{4}}.$$

4. Express as decimals $\frac{1}{27}$, and $\frac{5}{64}$; and as a vulgar fraction $\dot{6}\dot{7}6\dot{5}$.

5. Find the value of ·05625 of 1*l.* and ·38671875 of a cwt.; and reduce $4\frac{1}{2}d.$ to the decimal of half a crown.

6. Divide 138·4365 by 61·5 and 3·25 by 81; and *multiply 45·51* by ·0347.

7. *What is the* difference between the value of the

simple and compound interest on 275*l.* for 3 years at 5 per cent. per annum?

8. If I invest 3570*l.* in stock at 2 premium, which pays a dividend of 7 per cent., what income do I derive from it?

9. A grocer bought 40 lbs. of tea at 2*s.* 6*d.*, and 90 lbs. of coffee at 1*s.* 9*d.* per lb.; he sold the tea at 4*s.*, and the coffee at 1*s.* 6*d.*: what did he gain or lose by the transaction?

XV.

1. Find the L.C.M. of 34, 51, 85, 120, 170.

2. Simplify

$$2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \frac{1}{2}}}}}, \text{ and } \frac{12{\cdot}4 + {\cdot}064 - {\cdot}066}{{\cdot}002}.$$

3. Add together $\frac{1}{4}$ of $\frac{3}{5}$ of 5*s.* 4*d.*, $\frac{2}{3}$ of $\frac{5}{7}$ of 6*s.*, and $\frac{3}{7}$ of $\frac{2}{5}$ of 2*s.*

4. The rental of an estate is 1384*l.* 16*s.* per annum: what is the nett income derived from it after deducting an assessment of 2*s.* $9\frac{1}{2}$*d.* in the pound?

5. Two clocks are set together; one loses 7 seconds in 24 hours, the other gains 8 seconds in the same time: when will one be half an hour before the other?

6. Reduce $3\frac{3}{4}$*d.* to the decimal of 10*s.*, and divide the result by 12·5.

7. Find the cost of papering a room 21 ft. long, $16\frac{1}{2}$ ft. wide, $10\frac{1}{2}$ ft. high at $7\frac{1}{2}$ per square yard.

8. *Find the* square roots of 3452164 and of ·0001.

9. A person buys shares in a railway at 19*l.* 10*s.* when 15*l.* has been paid, and after paying a call of 10*l.* per share, sells them again at 32*l.* 9*s.*: how much per cent. does he gain?

10. If Moira coal costs 31*s.* per ton, and Wallsend coal 46*s.* per ton, how much do I gain or lose by buying 24 tons of Moira, supposing 3 tons of Wallsend to last as long as 4 tons of Moira?

XVI.

1. A can run 10 yards to B's 9: how many yards start must A give B in a mile to make an even race?

2. Find the G.C.M. of 180194 and 1973594.

3. Simplify $\dfrac{1-\frac{1}{5}\left(\frac{1}{2}+\frac{1}{7}\right)}{1-\dfrac{1}{2-\frac{1}{4}}}$ and $\dfrac{2\frac{3}{4}+2\frac{1}{2}\text{ of }\dfrac{7}{3\frac{4}{5}}-1\frac{2}{3}}{2-\frac{1}{2}\frac{5}{2}\frac{1}{8}}$.

4. Divide 1 by ·0025 and by ·00$\dot{2}\dot{5}$, and multiply 1000 by ·0001 and ·0001.

5. Add together ·0015 of ten shillings, 2·0615 of half a guinea, and 1·3357 of half a crown.

6. Find by Practice the value of 6156$\frac{1}{8}$ articles at 8*s.* 1$\frac{1}{2}$*d.* each.

7. A room 10 feet high, 20 feet long, and 16 feet wide; it contains a door 8 feet high by 4 feet wide, and two windows, each 5 feet by 3 feet: how many square yards of paper are required to cover the walls?

8. If I sell out 10000*l.* from the 3$\frac{1}{2}$ per cents. at 94$\frac{1}{2}$ *and buy 6 per cent.* debentures at 105, what will be the *difference in my income?*

9. Find the discount on 1025*l.* 12*s.* 6*d.* for six months at $5\frac{1}{6}$ per cent. per annum.

10. A, who has $\frac{3}{7}$ of a property, assigns $\frac{1}{5}$ of his share to his son; his son pays 7*l.* 10*s.* for income tax, at 6*d.* in the £: what is the annual value of the whole property?

XVII.

1. Reduce to its lowest terms $\frac{3}{4}\frac{5}{4}\frac{9}{6}\frac{7}{1}\frac{7}{2}\frac{5}{1}$, and find the L.C.M. of 2, 3, 4, 5, 8, 12, 72, 84.

2. Simplify the following fractions:—

$$(1)\ \frac{2\frac{1}{16}\frac{3}{8} \times \frac{1}{3}}{\frac{3}{8}\text{ of }\frac{5}{18} \div 5\frac{1}{6}} \div \frac{1}{8}\text{ of }1\frac{1}{16}.$$

$$(2)\ \cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\frac{1}{2}}}}}.$$

3. What will be the cost of paving a passage 15 yds. 2 ft. 6 in. long and 3 yds. 6 in. wide with tiles costing 18*s.* per square yard?

4. Add together $53\frac{1}{2}$, 36·875, $4\frac{5}{8}$, and $\frac{2}{3}$ of $7\frac{1}{2}$; and divide the result by ·01.

5. Find the value of 2·325*l.* + ·45*l.* − ·49*l.* − ·175 of 3*s.* 4*d.*, and reduce 3 lbs. 6 oz. to the decimal of 4 cwt. 2 lbs.

6. Find the square root of 1461681 and of 901·8009.

7. If 6 men can dig 5 acres in 8 days of 10 hours each, in how many days of 12 hours each can 4 men dig 6 acres?

8. At what rate of interest will 2560*l.* amount to 3328*l.* in 5 years?

9. If I sell out 1000*l.* from the 3 per cents. at 88, and buy shares at 110 bearing 5 per cent. interest, what difference do I make in my income?

10. A tradesman bought tea to the amount of 100*l.* at 2*s.* per pound, and retailed it at 4*s.* per pound; afterwards he bought tea to the same amount at 4*s.* per pound, but, in consequence of damage sustained in transit, he was compelled to retail it at half the cost price; what did he gain or lose on the double transaction?

XVIII.

1. A man has to place 10 stones in a row 10 yards apart from each other, returning for each stone separately to the place of starting: how far will he have to walk supposing that he deposits the first stone at the place of starting?

2. Simplify the following expressions:—

(1) $3-(\frac{1}{2}+\frac{1}{3})-(\frac{1}{5}+\frac{1}{6})$.

(2) $\left\{\frac{6}{5\frac{1}{2}}-\frac{2}{2+\frac{2}{2+\frac{2}{3}}}\right\}\quad(1\frac{5}{6}\times\frac{3}{2})$.

3. Divide 42·5 by ·017 and find the value of ·625 of 3 lbs. Avoirdupois.

4. If a man buys 560 tons of coal at 14*s.* 8*d.* per ton, and sells them at 11*d.* per cwt., how much will he gain by the transaction, and what will be his profit per cent. on his outlay?

5. Find the square root of 1·00060009.

6. *If 5 bricklayers build a wall 5 ft. high and 100 yards long in 3 days, how long will it take 7 men to build a wall 200 yards long and* $3\frac{1}{2}$ ft. high?

7. The gross yearly rental of an estate is 2100*l.*, and the deductions for rates and repairs amount to 25 per cent.; the owner sells the estate for 21 times the gross yearly rental, and invests the proceeds in $3\frac{1}{2}$ per cent. consols at 98: what does he gain or lose in his nett income?

8. A room is 10 ft high, 5 yds. 1 ft. long, and 3 yds. broad; it contains a door 8 ft. by 4, two windows each 5 ft. by 4, and a fireplace 6 ft. by 4 ft. 6 in.: how much paper will be required to cover the walls?

9. The total number of votes polled at an election was 3854, and the successful candidate polled 206 votes more than his opponent: what were the numbers polled on each side?

XIX.

1. A purse and its contents are together worth 2*l.* 4*s.*; the contents are worth ten times as much as the purse: what is the purse worth?

2. Simplify

(1) $$\frac{5}{5-\frac{5}{3-\frac{1}{2}}}+\frac{6}{6-\frac{6}{3-\frac{2}{3}}}.$$

(2) $$\frac{\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}}{\frac{2}{3}+\frac{2}{5}+\frac{2}{7}+\frac{2}{9}}.$$

3. By the reduction of the income tax from 6*d.* to 4*d.* in the pound a person saves 6*l.* 5*s.* a year: what is his income?

4. Find the sum of ·035 of 5*l.*, $\frac{3}{14}$ of $\frac{5}{9}$ of a guinea, and ·53 of 7*s.* 6*d.*; and express the result as the decimal of 10*l.*

5. The space between each of the posts of a telegraph by the side of a railway is 60 yards: find the rate of a train per hour which passes 11 such spaces in 50 seconds.

6. Multiply 1·0001 by ·0001, and divide the result by 10001.

7. What will be the income arising from the investment of 5308*l.* 6*s.* 8*d.* in the 3 per cents. at 91?

8. A wine merchant buys a hogshead containing 58 gallons of wine for 33*l.* 7*s.*; he pays 1*l.* 15*s.* for freight and expenses, and a shilling a gallon duty: at what price per gallon must he sell the wine so as to realise a profit of 20*l.*?

9. A invests 1200*l.*, B 2400*l.* in a certain business; the nett profits of the first year amount to 720*l.*, and are divided according to capital; how much do A and B severally receive and what is the rate of interest upon their investment?

XX.

1. A person bought 180 oranges at two a penny, and 180 at three a penny, and sold them at the rate of 5 for 2*d.*; what did he gain or lose by the transaction?

2. Find the L.C.M. of 5, 7, 16, 28, 48, and 21.

3. Add together $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$; subtract $\frac{5}{12}$ from $\frac{7}{9}$; divide $2\frac{1}{7}$ by $\frac{5}{9}$.

4. If 25 francs = 1*l.*, what fraction of a shilling is a franc?

5. If $\frac{1}{8}$ of a ship be worth 73*l.* 1*s.* 3*d.*, what portion of her will be worth 250*l.* 10*s.*?

6. If 10 men can reap 20 acres of corn in 4 days, how many men can reap 70 acres in 10?

7. Simplify

$$\frac{1\frac{3}{4} - \frac{7}{6} \text{ of } \frac{18}{28}}{\frac{5}{6} \text{ of } \frac{12}{20} \div 5\frac{1}{2}} \div \frac{1}{6}, \text{ and } \frac{\frac{1}{1\frac{1}{2}} + \frac{1}{2\frac{1}{2}} \cdot \frac{1}{3\frac{1}{2}} + \frac{14}{45}}{\frac{2}{15} + 1\frac{1}{3} + \frac{\frac{1}{5}}{\frac{1}{2} - \frac{2}{5}}}.$$

8. A sum of 1500*l.* is to be divided among 2 men, 3 women and 6 children; if a woman receives twice as much as a child, and a man twice as much as a woman what will each receive?

9. What sum of money will amount to 276*l.* 5*s.* in 15 months at 5 per cent. per annum simple interest?

10. How much stock at 93 will 697*l.* 10*s.* purchase? and when the 3 per cents. are at 75, what amount must be invested to produce an income of 120*l.* per annum?

11. Find the cost of carpeting a room 16 feet wide by 21 feet long, with carpet $\frac{3}{4}$ yard wide, at 5*s.* 3*d.* a yard.

XXI.

1. Find the present value of 1836*l.* due four years hence at 5 per cent. simple interest?

2. What will be the cost of carpeting a room 25 feet long and 13 feet 6 inches broad with carpet costing 10*s.* a square yard?

3. What is the rent of a field containing 112 acres 2 roods $29\frac{1}{2}$ poles at 2*l.* 12*s.* 10*d.* an acre?

4. Divide 1368·2394 by 2400·21 and by ·00240021, and add together ·14, ·0116, and ·32$\dot{5}$.

5. Simplify

(1) $\frac{1\frac{7}{9} \text{ of } \frac{27}{64}}{\frac{11}{12} \text{ of } 9\frac{9}{11}} \div \frac{4\frac{4}{7} \text{ of } \frac{21}{160}}{2\frac{5}{6} \text{ of } \frac{15}{34}}$

(2) $(\frac{1}{3} + \frac{1}{5} + \frac{6}{7} - \frac{4}{15}) \div (1 - \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5})$.

6. Find the value of $\frac{15\frac{3}{5}}{7\frac{4}{5}}$ of 1*l.* + $\frac{1}{5}$ of 140*l.* 10*s.* 6*d.* + $\frac{3}{5}$ of a guinea.

7. Find the G.C.M. of 1288, 1736, 104.

8. If 120 men can build a house 60 feet high in 15 days, how many will it take to build a house 55 feet high in 10 days?

9. Find the difference between the simple and the compound interest on 955*l.* at 6 per cent. for 4 years.

10. A train 88 yards long overtook a person walking along the line at the rate of 4 miles an hour, and passed him completely in 10 seconds; it afterwards overtook another person and passed him in 9 seconds: at what rate per hour was this second person walking?

11. A bankrupt paid 5*s.* 11*d.* in the pound, his assets being 71*l.* What were his debts?

XXII.

1. A, B, and C purchased a ship for 3300*l.*; B paid twice as much as A, and C paid four times as much as B: how much did each pay?

2. A garrison of 1000 men have provisions for 30 days; at the end of 10 days a reinforcement arrives; the provisions then last 5 days only: what was the number of the reinforcement?

3. Reduce 4*s.* 6$\frac{1}{2}$*d.* to the vulgar fraction of 1*l.*, and $\frac{1}{9}$ of a shilling to the fraction of 2*d.*

4. What is the value of

(1) $\frac{102}{480}$ of a day?

(2) $\frac{2}{3}$ *of* 17*l.* 17*s.* 6*d.* + $\frac{4}{5}$ of 1$\frac{2}{3}$ of 1*l.* + $\frac{5}{8}$ of a *crown?*

5. Simplify $\dfrac{(2+\frac{1}{5}\div(3+\frac{1}{7})}{(\frac{1}{2}-\frac{1}{3})\times(4-3\frac{3}{7})}$

6. Divide 2·975984 by 32·56, and 48 by ·04.

7. Express ·46̇6̇ as a vulgar fraction, and multiply 3·59 by 4·7.

8. Extract the square root of 3272869681.

9. A sum of money is borrowed at 5 per cent. per annum (simple interest); in 7 years it amounts to 810*l.*: what is the sum borrowed?

10. Find by Practice the cost of 75 cwt. 3 qrs. 21 lbs. of hops at 5*l.* 15*s.* 6*d.* the cwt.

11. What is the whole surface of the walls of a room 22 ft. 5 in. long, 18 ft. 4 in. broad, and 10 ft. 9 in. high; and the expense of painting the same at 8*d.* the square foot?

12. If 100 men in 6 days of 10 hours each can dig a trench 200 yards long, 3 yards wide, and 2 yards deep, in how many days of 8 hours long can 180 men dig a trench 360 yards long, 4 yards wide, and 3 yards deep?

13. What is the present worth 360*l.* 10*s.*, due in 11 months, at 6 per cent. per annum? Prove your answer.

XXIII.

1. A person left 30000*l.* to be divided amongst his three sons and three daughters in such a way that each of the daughters should have half as much as each of the two younger sons, and that the eldest son should have as much as a younger son and a daughter together: what was the share of each?

2. A room 11 ft. high and 17 ft. long requires 88 yards of paper $\frac{4}{6}$ of a yard wide to cover its walls: how many *square yards of* carpet will it require to cover its floor?

3. Simplify

(1) $5+\cfrac{5}{5+\cfrac{5}{5+\cfrac{5}{5+\frac{5}{6}}}}$.

(2) $\dfrac{3\frac{1}{3}+4\frac{1}{4}+5\frac{1}{5}}{1\frac{2}{3}+2\frac{1}{8}+2\frac{3}{5}} \times \dfrac{3\frac{4}{7}+4\frac{1}{16}+4\frac{5}{9}}{7\frac{1}{7}+8\frac{1}{8}+9\frac{1}{9}}$.

4. A square quadrangle contains 40092·0529 square feet: find the length of one of its sides.

5. Reduce 5*s.* $7\frac{1}{2}$*d.* to the decimal of 5*s.*; and find the value of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{8}$ of 3*l.*

6. Find by Practice the cost of 240 lbs. of tea at 2*s.* $1\frac{1}{2}$*d.* per lb. If 15 lbs. be damaged, what amount of profit will be gained by selling the remainder at 3*s.* per lb.?

7. Find the difference between the simple and compound interest on 1650*l.* for 2 years at $3\frac{1}{3}$ per cent.

8. Simplify

(1) $\dfrac{(\cdot075\times\cdot075)-(\cdot005\times\cdot005)}{(\cdot75-\cdot05)}$.

(2) $(\cdot3\dot{7}-1\cdot17)\times(\cdot052-1\cdot\dot{8})$.

9. A person's nett income after paying an income tax of 4*d.* in the pound, and also 4 per cent. on a mortgage of 5000*l.*, is 1570*l.*: what is his gross income?

10. A person invested 4706*l.* in the 3 per cents. at $90\frac{1}{2}$; a year afterwards, when the 3 per cents. had fallen to 89, he sold out one-half of the stock which was standing in his name and reinvested it in 5 per cent. foreign bonds at 52: what change did the reinvestment make in his income?

11. If 50 men dig a trench 120 yards long, 5 ft. broad, 10 ft. deep in three days of 10 hours each, in what time *will they dig* a trench 175 yards long, 4 ft. broad, and *8 ft. deep?*

XXIV.

1. Find by Practice the poor-rate on 1210*l.* 15*s.* at 3*s.* 4½*d.* in the £.

2. Find the G. C. M. of 3074 and 1653; and the L. C. M. of 14, 36, 84, 108, 144.

3. Simplify $\frac{9\frac{7}{8}-8\frac{6}{7}+6\frac{5}{6}-5\frac{4}{5}}{8\frac{9}{10}-7\frac{13}{15}+6\frac{7}{8}-5\frac{6}{7}}$ and $\frac{\cdot1\dot{3}\dot{6}\times7\frac{1}{3}}{\cdot4\dot{9}}$.

4. Extract the square root of 1706·5161.

5. If with a capital of 500*l.* a tradesman gain 50*l.* in 7 months, in what time will he gain 60*l.* 10*s.* with a capital of 385*l.*?

6. A certain 4½ per cent. stock is issued at a discount of 23 per cent.; what income should I get from 385*l.* invested in it?

7. If the number of third class passengers is to the number of first and second class together as 7 : 2, and the number of second class passengers is to the number of first class as 5 : 3, what fraction of the whole number of passengers will the first class amount to?

8. Find the cost of papering the walls of a room 10 ft. 8 in. wide, 19 ft. 4 in. long, and 9½ ft. high, with paper 2 ft. wide, at 2½*d.* a yard.

9. If 1 inch of rain on 4840 yards gives 18200 gallons of water, how many gallons will ·875 of an inch on a square mile give?

10. If the amount of property liable to income tax be 525 millions, at what rate per pound must the tax be taken to produce 17½ millions?

XXV.

1. The circumferences of two of the wheels of an engine being 25½ feet and 16½ feet respectively, how many

more times will one turn round than the other in $4\frac{1}{4}$ miles?

2. If the shadow of an upright pole 3 yards high be $2\frac{1}{2}$ yards long, what will be the height of a church spire which casts a shadow $82\frac{1}{2}$ yards long?

3. Reduce to simplest forms

$$\frac{3094}{30042} \text{ and } \frac{\frac{4}{7} \text{ of } 5\frac{3}{5}+\frac{1}{4} \text{ of } 3\frac{1}{3}}{2\frac{1}{5}+\frac{11}{12}} \div \frac{10\frac{1}{8}-8\frac{3}{4}}{6\frac{3}{4}-2\frac{1}{2}}.$$

4. Reduce $\frac{3}{4}$ of $2\frac{2}{3}$ of $\frac{1}{7}$ of 12*l.* 10*s.* 3*d.* to the fraction of 32*l.* 3*s.* 6*d.*; and 13*s.* $4\frac{1}{2}$*d.* to the decimal of 5*l.*

5. Find the value of 3·75 of half-a-crown + 4·4 of a guinea + 1·02$\dot{4}$ of 22*l.*

6. Find the length in yards of the side of a square enclosure which contains 4 acres 3 roods 24 poles.

7. What is the difference between the simple and compound interest on 125*l.* in two years, at 3 per cent.?

8. What would be the income derived from investing 1700*l.* 19*s.* 6*d.* in the Three-and-a-Half per Cents. at 95?

9. A plot of ground 50 yards long and 40 yards wide is to be laid with sods $1\frac{1}{2}$ feet long and 6 inches broad. If the sods cost 3*s.* per hundred, what will be the expense for turf alone?

XXVI.

1. Find the value of

(1) ·0125 of a sovereign. (2) ·$\dot{1}42857\dot{}$ of a cwt.

2. Simplify:

$$(1)\ 5-\cfrac{1}{1-\cfrac{1\frac{1}{4}}{1+\cfrac{3}{5+\frac{1}{3}}}};$$

$$(2)\ \left(1\tfrac{3}{7}-1\tfrac{1}{111}\right)\left(14-\tfrac{1}{8}\right)-\frac{1\frac{3}{5}+1\frac{1}{3}}{3\frac{1}{5}+2\frac{2}{3}}.$$

3. *Divide* ·01 *by* 50, ·0002, *and* ·09.

4. A bankrupt pays 5*s.* 9*d.* in the pound; if his assets were 500*l.* more, he would pay 6*s.* 5*d.*: what are his assets and his debts?

5. Find the present worth of 122*l.* 16*s.* due 7 months hence at 4 per cent.

6. Potatoes are bought at 10½*d.* the stone, and have to pay a duty of 1*l.* the ton; if they are sold at 1½*d.* the lb., what is the profit per cent.?

7. Find the compound interest on 5000*l.* for 4 years at 5 per cent.

8. Find the value of 3840 articles at 19*s.* 11¾*d.* each. Three purchasers divide them in the proportion of 3, 4, 5: what will each pay?

9. Extract the square root of ·196, $3\frac{2}{3} - 1\frac{8}{9}$.

10. I sell out 12500*l.* from the Three per Cents. at 96; I invest one-third of the proceeds in Egyptian Six per Cent. Bonds at 125, and the remainder in land which yields 2¼ per cent. net. What is my difference in income?

11. What quantity of Turkey carpet is required for a room 20 feet 4 inches long, and 18 feet 8 inches broad, allowing a margin of 2 feet 8 inches all round? Find also the number of tiles each 8 inches by 4 which will be needed to fill this margin.

XXVII.

1. Find by Practice the cost of 4 cwt. 2 qrs. 12 lbs. at 4*l.* 13*s.* 4*d.* per quarter.

2. Reduce $\frac{3}{7}$ of 1*d.* to the fraction of 17*s.* 6*d.*; and find what fraction 6 oz. 15 dwt. is of a lb. Troy.

3. Simplify:

(1) $\dfrac{7}{1 - \dfrac{2}{3 + \frac{3}{4}}}$; (2) $\left\{ 3\frac{3}{7} + \dfrac{2}{10\frac{1}{2}} - \left(\frac{5}{18} \text{ of } \frac{3}{7}\right) \right\} \div \frac{3}{7}$.

4. Find the value of ·3416̇ of 5*l.*; and multiply 17 acres 3 roods 10 perches by ·015.

5. In a town where the annual death rate is 2·5 per cent., the average number of deaths per week (counting 52 weeks to the year) is 228. What is the population of the town?

6. If 50 men can construct a fence 3 miles long in 60 days working 12 hours a day, how many hours a day must 120 men work to finish a fence 4 miles long in 40 days?

7. Find the cubical contents of a hall 111 feet long, 45 ft. 4 in. broad, and 40 feet high. What will be the cost of building it at 9½*d.* per cubic foot of its contents?

8. A person transfers 3000*l.* from a Four per Cent. Stock at 90 to a Three per Cent. Stock at 72. How much of the latter can he purchase, and what will be the difference in his income?

9. Supposing the length of the lower row of seats in the upper gallery of the theatre to be 140 feet, and each ascending row to contain 8 more seats than the one below it, except that the fifth and topmost row loses 16 spaces of 2 ft. 3 in. each between the windows, how many will the gallery hold, allowing 16 in. to each person?

XXVIII.

1. Find, by Practice, the value of 8640 articles at 2*s.* 5½*d.* each.

2. 3 tons of sugar are bought at 42*l.* per ton and sold at 5½*d.* per lb.: what is the gain on the transaction?

3. Simplify the fractions:

(1) $\dfrac{\frac{3}{35}\text{ of }2\frac{1}{3}+\frac{1}{3}\text{ of }\frac{12}{49}}{2-\frac{1}{45}\text{ of }29\frac{4}{7}}$;

(2) $\left(\frac{1}{4}\text{ of }\frac{16}{3}\text{ of }\frac{9}{7}\text{ of }\frac{35}{24}\right)\div\dfrac{13\frac{1}{3}-11\frac{2}{3}}{3\frac{1}{6}}$

4. What is the cost of papering a room 17 ft. 4 in. long, 12 ft. 8 in. broad, and 12 ft. high, with paper 2 ft. wide, at $7\frac{1}{2}d.$ the yard?

5. Add together $\cdot\dot{1}$ of a guinea, $\cdot 1\dot{6}$ of a shilling, and reduce the result to the decimal of a sovereign.

6. Find the square root of 42849, 119·0281.

7. A cistern 5 ft. deep, 16 ft. long, and 4 ft. 6 in. broad, is filled by one pipe in 45 minutes; in what time will a cistern 6 ft. deep, 20 ft. long, and 5 ft. broad be filled by three similar pipes?

8. What is the compound interest and amount of 2000*l.* for 3 years at 5 per cent.?

9. I have 13000*l.* in the Three and a Half per Cents., and sell out when they are at 98: how much money do I realise? and how much must I add to secure an equal income in the Three per Cents., buying in at 87?

10. The total rent of a farm is 1000*l.*; the following charges have to be made:—Land tax at 1 per cent.; poor rates at 7*d.* per £; other local rates at $11\frac{1}{4}d.$ in the £; tithe charge, 96*l.* 10*s.*; and insurance at 3*s.* per cent. What are the net receipts?

XXIX.

1. Simplify

(1) $12\frac{1}{12}+13\frac{5}{16}+17\frac{3}{9}+\frac{19}{144}$.

(2) $\frac{5\frac{1}{4}}{4\frac{1}{6}}\times\frac{1\frac{4}{11}}{2\frac{5}{21}}\div\frac{5\frac{3}{5}}{6\frac{5}{7}}$.

2. Find the value of $3\frac{7}{9}$ of 6*l.* 13*s.* $3\frac{3}{4}d.$, and the difference between $\frac{3}{11}$ of 5 furlongs and $\frac{5}{16}$ of $\frac{1}{4}$ mile.

3. Multiply ·416 by ·025, divide the product by 3·25, and reduce the result to a vulgar fraction.

4. *What* decimal of 5 cwt. is equal to ·1596 of a ton?

5. A room being 19 ft. 6 in. long, 15 ft. 3 in. wide, and 12 ft. 9 in. high, find the area of the floor, the volume of the room, and the cost of the carpet, which is $\frac{3}{4}$ yd. wide, at 5*s.* 6*d.* per yard.

6. If the simple interest on 216*l.* 15*s.* amounts to 37*l.* 18*s.* $7\frac{1}{2}$*d.* in $3\frac{1}{2}$ years, what is the interest on 119*l.* 10*s.* for $5\frac{1}{4}$ years at the same rate?

7. One field is three times as large as another, and a third is half as large again as the other two together. If the three fields together contain 52 acres 3 roods 30 poles, what is the size of each?

8. A person invests 3276*l.* in Three per Cent. Stock at 91; at the end of six months he receives the half-year's dividend, which he invests in the same stock, then at 90. What is his second half-year's dividend?

9. A and B having money in the proportion of 5 : 7, B pays $\frac{1}{2}$ of his money to A and receives back $\frac{1}{3}$ of what A then has, thus leaving B with 6*s.* more than A. How much had each at first?

10. A clothier gains 25 per cent. by selling cloth at 20*s.* 10*d.* per yard; but a bale of 80 yds. being damaged, he has to reduce the price by 10 per cent. What is now his profit on the bale, and his gain per cent.?

11. Three men and five women, working together, earn between them 4*l.* 10*s.* in a week; but if one man and one woman were absent for half the week, the joint earnings would be only 3*l.* 18*s.* What are the wages of each man and woman?

12. If 1 lb. of gold, 22 carats fine, be worth 46*l.* 14*s.* 6*d.*, in what proportion ought copper (regarded as worthless) to be mixed with pure gold to make a compound worth 2*l.* 11*s.* 11*d.* per oz.?

QUESTIONS AND EXERCISES.

ALGEBRAICAL.

I. EVALUATION.

1. If $a = 1$, $b = 3$, $c = 5$, $d = 0$, find the values of

(1) $a^2+2b^2+3c^2+4d^2$.

(2) $\dfrac{12a^3-b^2}{3a^2}+\dfrac{2c^3}{a+b^2}+\dfrac{a+b^2+c^3}{b^2-2bc+c^2}$.

2. When $a = 1$, $b = 2$, $c = 3$, $d = 4$, shew that

$\{d-(c-b+a)\}\ \{(d+c)-(b+a)\}$
$=d^2-(c^2+b^2)+a^2+2(bc-ad)$.

3. Find the value of

$\dfrac{a^2+b^2-c^2+2ab}{a^2-b^2-c^2+2bc}$ if $a=4$, $b=\frac{1}{2}$, $c=1$.

4. What is the value of

$$\frac{\dfrac{x}{1+x}+\dfrac{1-x}{x}}{\dfrac{x}{1+x}-\dfrac{1-x}{x}}$$ when $x=\frac{5}{7}$?

5. If $a=25$, $b=9$, $c=4$, $d=1$, find the value of

$3\sqrt{a}+2\sqrt{4b}-\sqrt{9c}+\sqrt{16d}$.

6. When $a=0$, $b=2$, $c=4$, $d=6$, find the value of

$$3\sqrt[3]{2b^3-a}+2\sqrt[3]{b^3+c^2+7}-\sqrt[3]{2(b+c)^2-(b+d)^2}.$$

7. If $a=1$, $b=-1$, $c=0$, $d=2$, find the value of

$$(ac-bd)\ (ad-bc)\ (ab-cd).$$

8. Evaluate $(1-ab)\ (1-ac)\ (1-bc)$, when $a=1$, $b=0$, $c=-1$.

9. If $a=0$, $b=2$, $c=-3$, $d=4$, find the value of

$$4\ (ad-bc)-\{(a-b)-(c-d)\}^2.$$

10. Evaluate $a-\{bd-(b-4c)\}-\frac{3}{4ab}(ab-bc+ad)^2$ if $a=2$, $b=-3$, $c=0$, $d=1$.

11. Find the value of $2a^2b^2c-3\{a-(b-c)^3\}^2$, when $a=-3$, $b=-2$, $c=-1$.

12. Evaluate $\frac{a+b}{a-3b}+\sqrt{2b-a}+\frac{b^3-a}{2b}$, when $a=-8$, $b=-2$.

Find the value of

$x^3+(x^2-42x+89)^{\frac{1}{2}}$, when $x=2$.

14. Evaluate

$$\frac{a}{b}-\left(\frac{1+a}{1-b}\right)^{\frac{1}{2}},\text{ when } a=\tfrac{1}{4},\ b=\tfrac{1}{5}.$$

15. Find the value of

$$p^0+mq^{-1}-r^{\frac{1}{2}},\text{ when } m=6,\ q=3,\ r=4.$$

16. If $a=3$, $b=4$, $c=9$, find the value of

$$\frac{2ab\sqrt{c-8}+\frac{1}{2}\sqrt{2(c-1)}}{\sqrt[3]{ac}+(a^2b-1)a-b}.$$

II. ADDITION AND SUBTRACTION.

1. Add together

(1) $4x^3-3x^2y+7xy^2-10y^3$
and $2x^3-5x^2y-xy^2+2y^3$.

(2) $x^3+x^2y-4xyz$, $y^3-3y^2z+3xy^2$,
$2z^3+3xz^2-3yz^2$, and $3x^2z-2xyz-z^3$.

(3) $bx^2+(a^2-ab)\,x+a^3$
and $(a-b)x^2+(a^2+ab)x-(a^3-b^3)$.

(4) $(b-c)x+(c-a)y+(a-b)z$
and $(2a-b)x+(3b-c)y+(4c-a)z$.

(5) $ax^3+(a+b)x^2-(2a-b)x+b$
and $bx^3-(a-c)x^2+(b+c)x-(a+b+c)$.

(6) $\frac{1}{2}a+\frac{2}{3}b-\frac{3}{4}c+\frac{1}{6}d$ and $\frac{2}{3}a-\frac{5}{6}b+c-\frac{1}{4}d$.

(7) $\frac{3}{5}(y+z)+\frac{5}{3}(z+x)$, $\frac{3}{4}(z+x)+\frac{2}{3}(x+y)$,
$\frac{7}{4}(x+y)+\frac{11}{2}(y+z)$.

2. Subtract

(1) $4x^3-3x^2-x+2$ from $7x^3-6x^2+2x-1$.

(2) $\frac{1}{2}x^4-\frac{3}{2}x-2\frac{1}{4}$ from $\frac{5}{2}x^4+\frac{1}{2}x^3+3x-1\frac{3}{4}$.

(3) $(b-c)x^2+(c-a)xy+(a-b)y^2$
from $ax^2+bxy+cy^2$.

(4) $2-(a+3)x+a^2x-5a^2x^3$
from $3a^2x-2a^2+x+1$.

(5) $a(x-y)+b(y-z)+c(z-x)$
from $a(x+y)+b(y+z)+c(z+x)$.

(6) $bx^m+(a+b)x^{m-n}-ax^{m-2n}$
from $ax^{m+n}+(a+b)x^m+2bx^{m-n}$.

(7) $(a-b)\sqrt{x^2+y^2}+c(a+x)^3$
from $(a+b)\sqrt{x^2+y^2}+(a+c)(a+x)^3$.

III. MULTIPLICATION.

1. Multiply

(1) $x^2+2xy-3y^2$ by $x^2-5xy+4y^2$.

(2) $1+x+x^4+x^5$ by $1-x+x^2-x^3$.

(3) $x^4+x^3y-x^2y^2+xy^3+y^4$ by $x-y$.

(4) $4ab-2ac$ by $6ab+3ac+8bcd$.

(5) $2a(x+y)-bz$ by $3b(x-z)+2ay$.

(6) $a^{2n}+a^nx^n+x^{2n}$ by $2a^n-2x^n$.

(7) a^m+b^m by a^n-b^n.

(8) $a^{\frac{3}{5}}-a^{\frac{1}{5}}b^{\frac{1}{5}}+b^{\frac{2}{5}}$ by $a^{\frac{1}{5}}+b^{\frac{1}{5}}$.

(9) $x^{\frac{1}{3}}-y^{\frac{1}{3}}$ by $x^{\frac{2}{3}}+x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}$.

(10) $x^{\frac{7}{3}}y^{\frac{1}{3}}-x^{\frac{4}{3}}y+x^{\frac{1}{3}}y^{\frac{7}{3}}$ by $x^{\frac{2}{3}}y^{-\frac{1}{3}}+x^{\frac{1}{3}}y^{\frac{2}{3}}$.

(11) $x+2y^{\frac{1}{2}}+3z^{\frac{1}{3}}$ by $x-2y^{\frac{1}{2}}+3z^{\frac{1}{3}}$.

(12) $2x^2-\frac{3}{4}x+1$ by $x^2-\frac{1}{2}x$.

(13) $x^3-\frac{x}{4}-3$ by $\frac{x}{3}-\frac{1}{2}$.

(14) $\frac{2a}{3}-\frac{3b}{4}$ by $\frac{4a^2}{9}+ab+\frac{9b^2}{16}$.

(15) $4a-6b\left(\frac{a}{2}+b\right)+3ab$ by $6b^2-4a$.

(16) $3ax^{-1}-(2a^{\frac{3}{2}}x^{-\frac{3}{2}}+1)$ by $(2a^{\frac{3}{2}}x^{-\frac{3}{2}}-1)-a^{\frac{1}{2}}x^{-\frac{1}{2}}$.

(17) $\frac{1}{8}x^m+\frac{1}{4}x^{m+1}y^{-1}+\frac{1}{2}x^{m+2}y^{-2}+x^{m+3}y^{-3}$
by $8x^2+16x^3y^{-1}+32x^4y^{-2}$.

(18) $x^3-(a+b+c)x^2+(bc+ca+ab)x-abc$
by $x-a$.

(19) $x^2-(a-b)x-ab$ by $x^2+(a-b)x-ab$.

2. Write down the products of

(1) $2x-3$, $2x+3$. (2) $x+\frac{a}{2}$, $x-\frac{a}{2}$, $x^2+\frac{a^2}{4}$.

(3) $x+y$, x^2-xy+y^2. (4) $x^{\frac{1}{2}}+x^{-\frac{1}{2}}$, $x-1+\frac{1}{x}$.

(5) $ax-b$, $a^3x^3+a^2bx^2+ab^2x+b^3$.

(6) $x^{2m}-x^my^n+y^{2n}$, $x^{2m}+x^my^n+y^{2n}$.

3. Find the continued product of

(1) $x+1$, $x+2$, $x+3$. (2) $10-x$, $x+1$, $x+4$.

(3) $2x-m$, $2x+n$, $x+2m$, $x-2n$.

(4) $3x-1$, $x^2+\frac{1}{9}$, $3x+1$.

(5) $(a+b+c)(a+b-c)(a+c-b)(b+c-a)$.

(6) $x-a$, $x+a$, x^2+ax+a^2, x^2-ax+a^2.

(7) $x^{\frac{1}{4}}+y^{\frac{1}{4}}$, $x^{\frac{3}{4}}-x^{\frac{1}{2}}y^{\frac{1}{4}}+x^{\frac{1}{4}}y^{\frac{1}{2}}-y^{\frac{3}{4}}$, $x+y$.

(8) $x^{\frac{1}{3}}+x^{-\frac{1}{3}}$, $x^{\frac{1}{3}}-x^{-\frac{1}{3}}$, $x^{\frac{2}{3}}+1+x^{-\frac{2}{3}}$.

4. Multiply x^3-ax^2+bx-c by x^2+mx+n, and arrange in brackets the coefficients of like powers of x.

5. Find the coefficient of x^3 in the product of

$x^3-bx^2-3abx+a^2$ and $2x^2-3ax-b^2$,

and the coefficient of x in the continued product of

$x-3m$, $x+2n$, $2x-n$, $3x+m$.

IV. DIVISION.

1. Divide

(1) $x^5-4x^3y^2-8x^2y^3-17xy^4-12y^5$
by $x^2-2xy-3y^2$.

(2) $x^6+2x^4-7x^3-16$ by x^3+2x^2+3x+4.

(3) $x^6-2a^3x^3+a^6$ by $x^2-2ax+a^2$.

(4) $a^5-a^3b^2+2a^2b^3-ab^4+b^5$ by a^2-ab+b^2.

(5) $a^2+ab+2ac-2b^2+7bc-3c^2$ by $a-b+3c$.

(6) $a^2+2ac-2bd-b^2+c^2-d^2$ by $a-b+c-d$.

(7) $a^3+b^3+c^3-3abc$ by $a+b+c$.

(8) $1+x^3-8y^3+6xy$ by $1+x-2y$.

(9) $x^4-9x^2-6xy-y^2$ by x^2+3x+y.

(10) $1-6x^5+5x^6$ by $1-2x+x^2$.

(11) $ax^3-(a^2+b^2)x^2+b^2$ by $ax-b$.

(12) $2abx^2-(3bc-2ad)xy-3cdy^2$ by $bx+dy$.

(13) $x^5-1-p(x^4-x)+q(x^3-x^2)$ by $x-1$.

(14) $m^3nx^4+(2m^2-3n^2)(m^2x^3y-3mnx^2y^2+9n^2xy^3)$
$-54mn^3y^4$ by $nx+2my$.

(15) $2a^{3n}-6a^{2n}b^n+6a^nb^{2n}-2b^{3n}$ by a^n-b^n.

(16) $prx^{m+n}+qrx^{2n}-(r^2-pq)x^n+p^2x^m-pr$
by rx^n+p.

(17) $2a^{\frac{3}{2}}-3a^{\frac{1}{2}}b^{\frac{1}{2}}-2a^{\frac{1}{2}}b^{\frac{1}{2}}+3b$ by $a^{\frac{1}{2}}-b^{\frac{1}{2}}$.

(18) $x+x^{\frac{1}{2}}y^{\frac{1}{2}}+y$ by $x^{\frac{1}{2}}-x^{\frac{1}{4}}y^{\frac{1}{4}}+y^{\frac{1}{2}}$.

(19) $\frac{1}{4}a^3+\frac{3}{8}a^2x-2x^3$ by $\frac{1}{2}a+x$.

(20) $a^2+\frac{a}{x}-\frac{x}{a}-x^2$ by $a+\frac{1}{a}+\frac{1}{x}+x$.

(21) $\frac{a^4}{2}+1\frac{1}{2}+2a$ by $\frac{a^2}{2}-a+1\frac{1}{2}$.

(22) $-\frac{5}{9}a^2+\frac{1}{3}\frac{1}{3}ab-\frac{1}{3}\frac{0}{0}ac+\frac{1}{4}\frac{5}{6}b^2+25bc$ by $\frac{2}{3}a-5b$.

(23) $\frac{1}{2}x^3+x^2+\frac{3}{8}x+\frac{3}{4}$ by $\frac{1}{2}x+1$.

(24) $\frac{x^4}{a^2}-\frac{x^3}{a}-\frac{a^3}{x}+\frac{a^4}{x^2}$ by $x^2-2ax+a^2$.

(25) $(2x^2-7x+6)a^4+(x^2-7x+7)a^3-(3x+4)a^2$
$-(x+7)a-2$ by $(x-2)a^2-3a-1$.

2. Divide

(1) a by $1+2x$ to four terms.

(2) a by $1+5x$ to five terms.

(3) $x-a$ by $x-b$ to three terms.

3. Divide $8x^3-y^3$ by $(2x-y)^2$, writing the remainder as a fraction.

4. Divide x^3 by $1-x^2$ to three terms. What is the difference between your answer and the true one?

5. Find the difference between $\frac{1}{1-x}$ and

$$1+x+x^2+x^3+\&c.+x^{n-1}.$$

6. Find the value of $\frac{x}{1-x}$ to four terms, and the remainder, when n terms of the quotient have been taken.

7. Write down the quotients of

(1) $49x^2-25y^2$ divided by $7x-5y$.

(2) $\frac{x^2}{4y^2}-\frac{4y^2}{x^2}$ „ $\frac{x}{2y}+\frac{2y}{x}$.

(3) $a^{2m}b^{4n}-c^{6p}$ „ $a^mb^{2n}-c^{3p}$.

(4) $a^3x^3+b^3y^3$ „ $a^2x^2-abxy+b^2y^2$.

(5) $27x^3-a^3$ „ $3x-a$.

(6) x^6-a^6 „ $x-a$, $x+a$, x^3-a^3, x^3+a^3, severally.

(7) $4x-\frac{y^2}{4}$ by $2x^{\frac{1}{2}}-\frac{y}{2}$. (8) $\frac{x^3}{27}+y^3$ by $y+\frac{x}{3}$.

(9) $\frac{a^3}{8}-b^3$ by $b-\frac{a}{2}$. (10) $a^{\frac{3}{4}}-b^{\frac{3}{4}}$ by $a^{\frac{1}{4}}-b^{\frac{1}{4}}$.

8. *Shew that* a^n-b^n is always divisible by $a-b$.

9. Under what conditions will a^n+x^n divide by $a+x$ without remainder? Write down the first three and the last three terms of the quotient.

10. Prove that $a^{mn}-b^{mn}$ is divisible by both a^m-b^m and a^n-b^n, and give the first and second terms and the ast and last but one in each quotient.

V. SIMPLIFICATION.

1. Resolve into their factors

(1) $a^2x^2y-b^2xy^2$. (2) $a^2x^2-a^2y^2$.

(3) ax^2-a. (4) x^4-81a^4.

(5) $4x^4-36y^4$. (6) $16x^{12}-y^4$.

(7) $x^{16}-y^{16}$. (8) $8a^3+27$.

(9) $8x^6+\frac{1}{x^3}$. (10) $a^3b^3c^3-64b^3$.

(11) $16x^4-y^4$. (12) $32x^5+y^5$.

(13) $x^{12}-y^{12}$. (14) $x^2+7x+10$.

(15) x^2+6x-7. (16) $x^2+5x-14$.

(17) $2x^2-3x-5$. (18) $5x^2+8xy-21y^2$.

(19) $6a^2x^2-7ax-3$. (20) $a^2-b^2+c^2-2ac$.

(21) $m^2-n^2-p^2+q^2+2(mq-pn)$.

(22) $\frac{3ad}{b}-cd-1+\frac{bc}{3a}$.

2. Arrange according to descending powers of x, col*lecting the coefficients* of each power into brackets,

$$x^3-ax-c^2x^2-bx+bx^3-cx^2+a^2x^3-x^3-cx.$$

3. Subtract

$(a+y)x^3-(2a^2-y^2)x^2-2xy^3$ from $a^3y-(2x^2-a^2)y^2+ax^3$, and arrange your answer in descending powers of y, collecting coefficients.

4. Arrange according to powers of x, collecting coefficients,

$$3(a^2-2x)(a-x^3)+2x(a-bx)^2.$$

5. Simplify

(1) $(3a-7b+4c)+(a-4b+2c)-(2a-3b+2c)$.

(2) $1-\{1-(1-4x)\}-\{2-(-4+5x)\}$.

(3) $a^2-(b^2-c^2)-\{b^2-(c^2-a^2)\}+\{c^2-(b^2-a^2)\}$.

(4) $3(x+a)(y-b)-\{-a[b-(c-d)]\}$.

(5) $3a-\frac{1}{2}[b+\{2a-(b-x)\}]$.

(6) $3bx(c-d)-\frac{1}{2}\{4ad-6(a-b)(c-d)\}x$.

(7) $4\{(2ab^2c^4)^4-2ab^2c(2ab^2c^5)^3\}$.

(8) $(x+y)^2-2x(3x+2y)-(y-x)(-x+y)$.

(9) $3a-\{2a-(3a-b)^2\}+3a\left\{2b-3a-\dfrac{b^2}{3a}\right\}$.

(10) $l-m-3\left[\left\{l-m+lm\left(\frac{1}{l}+\frac{1}{m}\right)\right\}^2-4l^2\right]$.

(11) $1-\{1-(1+x)^2\}+3\left\{2-(x-1)^2-\dfrac{2x^3}{3}\right\}$.

(12) $2\{a(a-b)-(a+b)^2\}(a^2-2b^2)$.

(13) $3b\{a(a-b)(3a+b)-(a-b)^3\}$.

6. Prove the following equivalents:

(1) $x(a^2-\overline{ax-x^2})+ax(x-\overline{2b-3c})$
$-a(ax-\overline{2bx-3cx})=x^3$.

(2) $a^2-\{2(a+b)+c\}c+2a\{\frac{1}{2}(a+b+c)-\frac{1}{4}b\}$
$=2a(a-\frac{1}{2}c)+\frac{1}{2}b(a+2b)-(b+c)^2.$

(3) $(p+q+r)^2-(p^2+q^2+r^2)=p(q+r)+q(p+r)$
$+r(p+q).$

7. Shew that if $a=\frac{b^2}{c}$, and $d=\frac{c^2}{b}$, then

$$(a+b)(c+d)=(b+c)^2.$$

8. Prove that if $p=1-q$,

$$\frac{2p^2-2p+1}{p^2-p}=\frac{p}{p-1}+\frac{q}{q-1}.$$

9. Give equivalent forms for

(1) $x^{\frac{m}{n}}$. (2) $x^{-\frac{1}{p}}$. (3) $(xy)^0$. (4) $\frac{x^5}{y^5}$.

(5) $(a^m)^n$. (6) $\frac{a^{m+1}}{a^{m-1}}$. (7) $\frac{x^{n-1}}{x^{n-2}}$. (8) $x(xy)^{-1}$.

(9) $\frac{a^{-}}{b^{-n}}$. (10) $(a^{-p})^{-q}$.

10. Simplify

(1) $(a^{\frac{3}{2}}b^{-\frac{1}{2}})^{\frac{2}{3}}+(a^{-\frac{1}{2}}b^2)^{\frac{1}{3}}$. (2) $[\{a^2(ab^{\frac{1}{2}})^{\frac{2}{3}}\}^{\frac{1}{2}}]^3$.

(3) $\{ab^2(ab^3)^{\frac{1}{2}}(ab^4)^{\frac{1}{3}}\}^{\frac{2}{5}}$.

11. What are the meanings of a^m, a^{m-n}, $a^{\frac{m}{n}}$, and a^{mn}?

VI. GREATEST COMMON MEASURE.

1. Find the G.C.M. of

(1) $12a^2x^3y^4$ and $14a^3xy^5$.

(2) a^2x^2-2ax, $a^2x^3-3a^2$.

(3) $2ax-2ay$, $3bx-3by$.

(4) $8a^2x^3+12a^3x^2$ and $12x^5+18ax^4$.

(5) $9x^2-25$ and $9x^2+3x-20$.

(6) $12x^4-108x^3$ and $16x^4-48x^3$.

(7) $4x^3-xy^2$ and $4x^3-10x^2y+4xy^2$.

(8) $a^4-2a^3x+a^2x^2$ and $a^4-2a^2x^2+x^4$.

(9) a^4-x^4, and $a^3+a^2x-ax^2-x^3$

(10) $6a^6-6a^3x^3$ and $8a^6-8a^5x$.

(11) $6a^2+7ax-3x^2$ and $6a^2+11ax+3x^2$.

(12) $3x^2+16x-35$ and $5x^2+33x-14$.

(13) $6a^3-6a^2y+2ay^2-2y^3$ and $12a^2-15ay+3y^2$.

(14) $7x^2-12x+5$ and $2x^3+x^2-8x+5$.

(15) $2a^3+2a^2y-3ay^2-3y^3$ and $3a^2+ay-2y^2$.

(16) $12x^6+2x^3-2$ and $16x^7-12x^4-10x$.

(17) x^3+3x^2+3x+2 and x^3-x^2-x-2.

(18) $2x^4+11x^3-13x^2-99x-45$ and $2x^3-7x^2-46x-21$.

(19) $3x^3-22x-15$ and $5x^4+x^3-54x^2+18x$.

(20) $5x^3-12x^2+19x-6$ and $3x^4-23x^3+43x^2-51x$.

(21) $3x^4+7x^3-15x^2+9x-2$ and $6x^3+23x^2-39x+14$.

(22) $x^4-8x^3+21x^2-20x+4$ and $2x^3-12x^2+21x-10$.

(23) $y^4+m^2y^2+m^4$ and $y^4+my^3-m^3y-m^4$.

(24) $3x^5-10x^3+15x-8$ and $2x^4-4x^2+2$.

(25) $x^3-3x^2+7x-21$ and $2x^4+19x^2+35$.

(26) $x^4+5x^3-7x^2-9x-10$ and $2x^4-4x^3-3x^2+8x-4$.

(27) $x^4+x^3-2x^2+3x-3$ and $x^5-4x^3-2x^2+3x+2$.

(28) x^3-8x+3 and x^6+3x^5+x+3.

(29) $a^2-4x^2+12x-9$ and $a^2+2a-4x^2+8x-3$.

2. Explain what is meant by greatest common measure.

3. Prove the rule for finding the G.C.M. of two quantities, when neither of them has any simple factor.

VII. LEAST COMMON MULTIPLE.

1. Find the L.C.M. of

(1) axy and $a(xy-y^2)$.

(2) $x^2(x-y)$ and $x(x-y)^2$.

(3) $2a^2+2axy$ and $axy+x^2y^2$.

(4) a^2-ab, a^2b, ab^2, $ab-b^2$.

(5) b^2+bxy and $bxy-x^2y^2$.

(6) $2a^2(a+x)$, $4ax(a-x)$, $6x^2(a+x)$.

(7) xy, $x-y$, y^3-x^2y.

(8) $6ax^3+6bx^3$, $a^2x^2-b^2x^2$, $15x$.

(9) $3x^2y+6xy^2$ and $2x^2-8y^2$.

(10) $4(a^2-a)$ and $6(a^2+a)$.

(11) x, x^2, $(x+1)^2$, x^2-1, $(x-1)^2$.

(12) x^3+y^3, x^3-y^3, $3(x-y)$.

(13) $3(1-x^2)$, $6(1-x)^2$, $5(x+x^2)$.

(14) $a-x$, $a+x$, a^2-x^2, a^3+x^3.

(15) $4(1-x)^2$, $8(1-x)$, $8(1+x)$, $4(1+x^2)$.

(16) $4ab(a^2-b^2)^3$ and $6a^3b^3-6a^2b^3$.

(17) x^2-1, x^2+x-2, x^2-5x-6.

(18) a^2-1, a^2+2a-3, a^3-7a^2+6a.

(19) $x^3+x^2y+xy^2+y^3$ and $x^3-x^2y+xy^2-y^3$.

(20) $2x^3+5x^2y+5xy^2+3y^3$ and $2x^3+x^2y-xy^2+3y^3$.

(21) x^3-8x^2+7, x^3+7x-8.

(22) $3x^3-15ax^2+a^2x-5a^3$ and $6x^4-25a^2x^2-9a^4$.

2. Find the G.C.M. and the L.C.M. of

(1) $(2a^2bx+2abx^2)^2$ and $(6a^2b^2-6b^2x^2)^3$.

(2) $2x^3-9x^2y+11xy^2-3y^3$ and $4x^3-4x^2y-5xy^2+3y^3$.

3. Find all the quantities whose L.C.M. is x^3-4xy^2.

4. Explain what is meant by a common multiple, and what by the least common multiple of two or more quantities.

5. State how the least common multiples of quantities may most conveniently be found when the quantities are resolvable into factors.

6. Establish a rule for finding the L.C.M. of two quantities when their factors are not discoverable.

7. Shew that every common multiple of two quantities is also a multiple of their L.C.M.

VIII. FRACTIONS.

1. Reduce to lowest terms

(1) $\frac{3cx^2y-b^2cy}{acy+2c^2y}$.

(2) $\frac{3x^2y^2+4xy^3}{5x^3y^2-4x^3y}$.

(3) $\frac{14x^2-7xy}{10ax-5ay}$.

(4) $\frac{(a^2+ax)^2}{(ax^2+x^3)^3}$.

(5) $\frac{ax^2y-axy^2}{bx^4y-bx^2y^3}$.

(6) $\frac{3x^2-3xy}{4(x^2-y^2)}$.

(7) $\frac{x^4+x^2y}{x^4-y^2}$.

(8) $\frac{49a^3-98a^2x}{7a^4-28a^2x^2}$.

(9) $\frac{a^3-b^3}{(a-b)^2}$.

(10) $\frac{(x-y)^3}{3x^2y-3xy^2}$.

(11) $\frac{b^2-a(2b-a)}{a^2-b^2}$.

(12) $\frac{a^2-ab-12b^2}{a^2+5ab+6b^2}$.

(13) $\frac{x^2+3x-10}{3x^2+2x-16}$.

(14) $\frac{x^2-3x-4}{2x^2-9x+4}$.

(15) $\frac{3x^2+16x-35}{5x^2+33x-14}$.

(16) $\frac{a^2+(m-n)a-mn}{a(a+m)-n(a+m)}$.

(17) $\frac{acx^2+(ad-bc)\,x-bd}{a^2x^2-b^2}$.

(18) $\frac{abyx-bcx+ady-cd}{b^2x^2+2bdx+d^2}$.

(19) $\frac{a^2+b^2-c^2+2ab}{a^2-b^2-c^2+2bc}$.

(20) $\frac{a^2x^2-(a+y)\,abx+ab^2y}{x^3-(a+1)\,bx^2+ab^2x}$.

(21) $\frac{x^3-39x+70}{x^2-3x-70}$.

(22) $\frac{x^3+x^2+3x-5}{x^2-4x+3}$.

(23) $\dfrac{5x^3-12x^2+7x}{5x^3-8x^2+x+2}$. (24) $\dfrac{x^3-3x+2}{x^3+4x^2-5}$.

(25) $\dfrac{6x^3-x^2y-18y^3}{8x^3-16x^2y+9y^3}$.

(26) $\dfrac{x^3-2x^2-13x-10}{x^4+7x^3-56x^2-20x}$.

(27) $\dfrac{a^2b+bcd+ab^2+acd}{a^2b-bcd+ab^2-acd}$.

2. Multiply together

(1) $\dfrac{2a-b}{a+b}$, $\dfrac{2a-2b}{2a+b}$, $\dfrac{a^2+2ab+b^2}{4a^2-b^2}$.

(2) $\dfrac{x^2+x-2}{x^2-2x-3}$, and $\dfrac{x^2-x-2}{x^2+2x-3}$.

(3) $3ab$, $\dfrac{2a+1}{a-ab}$, $\dfrac{a^2-b^2}{b^2-4a^2b^2}$.

(4) $\dfrac{2ax+3a^2}{4x^2-3ax}$, $\dfrac{4ax^3-3a^2x^2}{a^2x^2-a^4}$, $\dfrac{a^2-x^2}{2x^2+3ax}$.

3. Divide

(1) $\dfrac{2a^2}{m^2n-mn^2}$ by $\dfrac{2m^2}{a^2n-an^2}$.

(2) $\dfrac{4a^2-4ax}{(a+x)^2}$ by $\dfrac{a^2-x^2}{4ax+4x^2}$.

(3) $\dfrac{a^2+ab+ac+bc}{a^2-ac+ad-cd}$ by $\dfrac{a+b}{a+d}$.

(4) $\dfrac{x^3-27}{(x+2)^3}$ by $\dfrac{(x-3)^3}{x^3+8}$.

4. Simplify

(1) $\dfrac{8a^2b}{c} \times \dfrac{c^2d}{8a^3} \times \dfrac{4ab}{cd} \times \dfrac{bcd-cd^2}{6b^2-6bd}$

(2) $\dfrac{a^2+2ax+x^2}{a^2-x^2} \times \dfrac{ax-x^2}{a^2+ax} \times \dfrac{ax+x^2}{a^2-ax}$.

(3) $\dfrac{a+b}{c+d} \div \dfrac{e-f}{g-h}$. (4) $\dfrac{l^3+m^3}{l^2-m^2} \div \dfrac{l^2-lm+m^2}{l-m}$.

(5) $\dfrac{a^4-x^4}{a^2-2ax+x^2} \div \dfrac{a^2x+x^3}{a^3-x^3}$.

(6) $\dfrac{a}{a+b} \times \dfrac{a^2-ab}{b} \times \dfrac{ab+b^2}{a-b} \div \dfrac{a^2-b^2}{(a+b)^2}$.

(7) $\dfrac{x^3-a^3}{x^3+a^3} \times \dfrac{x+a}{x-a} \div \dfrac{x^2-ax+a^2}{x^2+ax+a^2}$.

(8) $\dfrac{x^3+a^3}{2ax-x^2} \times \dfrac{x^3}{x^2-ax+a^2} \div \dfrac{4a^2x^2+2ax^3}{4a^2-x^2}$.

(9) $\dfrac{m^3-n^3}{m+n} \times \dfrac{m^2-n^2}{m^2+mn+n^2} \div 2(m-n)$.

5. Add together

(1) $\dfrac{a}{b}, \dfrac{b}{ax}, \dfrac{a}{axd}$. (2) $\dfrac{x+2}{3x-6}, \dfrac{x+3}{2x-4}$.

(3) $\dfrac{ad+acx}{bc+acx}, \dfrac{bc-acx}{ad-acx}$.

(4) $\dfrac{1}{(1+x)^2}, \dfrac{1}{(1-x)^2}, \dfrac{2}{1-x^2}$.

6. Subtract

(1) $\dfrac{a-x}{a+x}$ from $\dfrac{a}{x}$. (2) $\dfrac{x}{a^2-ax}$ from $\dfrac{a}{ax-x^2}$.

(3) ax from $\dfrac{2ax+a^2x^2+2x}{2+ax-2x^2}$.

(4) $\dfrac{x^3+y^3+(x+y)^2}{x+y}$ from $\dfrac{(x-y)^3+x^3-y^3}{x-y}$.

7. Simplify

(1) $\frac{a-b}{ab}+\frac{a-c}{ac}+\frac{b-c}{bc}$.

(2) $\frac{2p-q}{pq}+\frac{2r-p}{pr}+\frac{2q-r}{qr}$.

(3) $\frac{7x-10}{5}-\frac{3x-7}{6}=\frac{27x-30}{30}$.

(4) $\frac{a}{x^n}-\frac{c}{x^{n-1}}+\frac{d}{x^{n-r-s}}$. (5) $\frac{ax^{\frac{1}{2}}}{x^{-\frac{3}{2}}}+\frac{bx^{-\frac{1}{2}}}{x^{-\frac{5}{2}}}+\frac{x^{\frac{5}{2}}}{c^{-1}x^{\frac{1}{2}}}$.

(6) $\frac{2x}{x^2-4}-\frac{5x}{3x^2+x-10}$.

(7) $\frac{1}{2x^2+x-1}+\frac{1}{3x^2+4x+1}$.

(8) $\frac{x^2-2}{x-1}+\frac{2x-1}{x+3}+\frac{2x+1}{x}$.

(9) $\frac{a}{a+b}+\frac{b}{a-b}-\frac{a^2+b^2}{a^2-b^2}$.

(10) $\frac{1}{x-1}+\frac{x}{1+x}-\frac{x^2}{1-x^2}$.

(11) $2-\frac{x^2-y^2}{x^2+y^2}+\frac{x^2+y^2}{x^2-y^2}$.

(12) $\frac{1}{a-b}-\frac{ab}{a^3-b^3}-\frac{a^2b^2}{a^5-b^5}$.

(13) $\frac{a-b}{b}\cdot+\frac{2a}{a-b}-\frac{a^3+a^2b}{a^2b-b^3}$.

(14) $\frac{1}{x-1}-\frac{1}{2(x+1)}-\frac{x+3}{2(x^2+1)}$.

(15) $\frac{m}{n}-m\frac{p}{n^2+np}-\frac{1}{n}\frac{mn-2mp}{n-p}$.

(16) $\frac{1}{4a^3(a-x)}+\frac{1}{4a^3(a+x)}+\frac{1}{2a^3(a^2+x^2)}$.

(17) $\dfrac{x+1}{2x-1}-\dfrac{x-1}{2x+1}-\dfrac{1-3x}{x(1-2x)}$.

(18) $\dfrac{3a}{(a-2x)^2}+\dfrac{2a+x}{(a+x)\ (a-2x)}-\dfrac{5}{a+x}$.

(19) $\dfrac{1}{x^2}+\dfrac{x-1}{x^2+1}-\dfrac{1}{(x^2+1)^2}$.

(20) $\dfrac{m}{m+n}+\left(\dfrac{n}{m+n}\right)^2+\dfrac{mn}{(m+n)^2}$.

(21) $\left(\dfrac{p}{a}-\dfrac{q}{b}\right)\dfrac{r}{c}+\left(\dfrac{p}{a}-\dfrac{r}{c}\right)\dfrac{q}{b}+\left(\dfrac{q}{b}-\dfrac{r}{c}\right)\dfrac{p}{a}$.

(22) $\dfrac{1}{(a-b)\ (a-c)\ (x+a)}-\dfrac{1}{(a-b)\ (b-c)\ (x+b)}$
$+\dfrac{1}{(a-c)\ (b-c)\ (x+c)}$.

(23) $\dfrac{1}{1-x+x^2}-\dfrac{1}{x^{-1}-1+x}+\dfrac{1}{x^{-2}-x^{-1}+1}$.

(24) $\dfrac{1}{(2x-1)^2}+\dfrac{1}{(2x+1)\ (1-2x)}$
$+\dfrac{1}{2x^2\ (2x-1)\ (1-4x^2)}$.

8. Multiply $\dfrac{a^{\frac{1}{2}}+b^2}{2}+\dfrac{3a^{\frac{1}{2}}}{4}$ by $18\ (a^{\frac{1}{2}}+b+c^2)$.

9. Divide

(1) $1-\dfrac{x^2}{x^2+a^2}$ by $1+\dfrac{a^2}{x^2-a^2}$.

(2) $\dfrac{a+x}{a-x}+\dfrac{a-x}{a+x}$ by $\dfrac{a+x}{a-x}-\dfrac{a-x}{a+x}$.

10. Simplify

(1) $\left\{\dfrac{1}{3}+\dfrac{2x}{3(1-x)}\right\}\times\left\{\dfrac{3}{4}-\dfrac{3x}{2(1+x)}\right\}$.

(2) $\frac{3}{xy} \times \left(x + \frac{xy}{3}\right) \times \left(1 - \frac{3}{3+y}\right)$.

(3) $\left(x + \frac{2x}{x-3}\right) \div \left(x - \frac{2x}{x-3}\right)$.

(4) $\left(\frac{x}{x+y} + \frac{y}{x-y}\right) \div \left(\frac{x}{x-y} - \frac{y}{x+y}\right)$.

(5) $\left(1 + \frac{1}{x}\right) \div \left(x - \frac{1}{x}\right) \times \left(1 - \frac{1}{x}\right)^2$.

(6) $\frac{y^2(xy^{-1}-1)^2}{x(1+x^{-1}y)^2} \times \frac{y^2(x^{-2}+y^{-2})}{x(xy^{-1}-x^{-1}y)} \div \frac{1-x^{-1}y}{xy^{-1}+1}$.

11. Shew that

(1) $1 + \frac{a^2+b^2-c^2}{2ab} = \frac{(a+b+c)\,(a+b-c)}{2ab}$.

(2) $1 - \frac{a^2+b^2-c^2}{2ab} = \frac{(a-b+c)\,(b-a+c)}{2ab}$.

(3) $\frac{a+c}{(a-b)\,(x-a)} - \frac{b+c}{(a-b)\,(x-b)} = \frac{x+c}{(x-a)\,(x-b)}$.

(4) $\frac{a^2-(b-c)^2}{(a+c)^2-b^2} + \frac{b^2-(a-c)^2}{(a+b)^2-c^2} + \frac{c^2-(a-b)^2}{(b+c)^2-a^2} = 1$.

12. Reduce

(1) $\frac{2x}{x-\frac{1}{3}}$.

(2) $\frac{3\frac{1}{4} - \frac{1}{3}(a-2)}{1\frac{1}{12} + (a-\frac{3}{2})}$.

(3) $\dfrac{\dfrac{a^3+b^3}{a^2-b^2}}{\dfrac{a^2-ab+b^2}{a-b}}$

(4) $\frac{1-\frac{1}{2}\{1-\frac{1}{3}(1-x)\}}{1-\frac{1}{3}\{1-\frac{1}{2}(1-x)\}}$.

(5) $\dfrac{x+2}{1 + \dfrac{x}{2x-x^2+3}}$.

(6) $\dfrac{\frac{1}{x} + \frac{1}{xy^3}}{y-1+\frac{1}{y}}$.

(7) $\dfrac{\dfrac{x-1}{x}}{x - \dfrac{1}{x+1}}$.

(8) $\dfrac{\dfrac{m}{m+n}+\dfrac{n}{m-n}}{\dfrac{2m}{m-n}-\dfrac{2n}{m+n}}$.

(9) $\dfrac{\dfrac{a+b}{a-b}-\dfrac{a+b}{a+b}}{\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}}$.

(10) $\dfrac{\dfrac{1}{1+x}+\dfrac{x}{1-x^2}}{\dfrac{1}{1-x^2}-\dfrac{x}{1+x}}$.

(11) $\dfrac{a+\dfrac{b-a}{1+ab}}{1-a\,\dfrac{b-a}{1+ab}}$

(12) $\dfrac{n\left(1+\dfrac{m-n}{m+n}\right)}{m\left(1-\dfrac{m+n}{m-n}\right)}$.

(13) $\dfrac{x+1}{\dfrac{1}{x+1}+\dfrac{1}{x-1}+\dfrac{2}{1-x^2}}$

(14) $\dfrac{1}{x+\dfrac{1}{x+\dfrac{1}{x}}}$.

(15) $a+\dfrac{1}{b+\dfrac{1}{a+\dfrac{1}{b}}}$.

(16) $\dfrac{2a}{1+\dfrac{2a}{1+\dfrac{2a}{1+2a}}}$.

(17) $\dfrac{1}{x-1+\dfrac{1}{1+\dfrac{x}{4-x}}}$.

13. Reduce $a-b+\dfrac{1}{1+\dfrac{b}{4a}}$ to a single fraction.

14. Shew that $\dfrac{1}{1+\dfrac{1}{x+\dfrac{1}{x}}}=\dfrac{1+x^2}{1+x+x^2}$.

15. Shew that

(1) If $x=\dfrac{am-bn}{m+n}$ and $y=\dfrac{bm+an}{m-n}$,

then $\dfrac{ay-bx}{(a+b)n}=\dfrac{(a+b)m+(a-b)n}{m^2-n^2}$.

(2) $\dfrac{a^n}{2na^n-2nx}+\dfrac{b^n}{2nb^n-2nx}=\dfrac{1}{n}$ when $x=\dfrac{a^n+b^n}{2}$.

(3) If $p=\dfrac{1}{q}$, then $\left(\dfrac{p+1}{q+1}\right)^2=\left(\dfrac{p-1}{q-1}\right)^2$.

(4) If $\dfrac{a}{b}=\dfrac{c}{d}$, $\dfrac{a+b+c+d}{c+d}=\dfrac{b+d}{d}$.

(5) If $\dfrac{m}{n}=\dfrac{p}{q}$, then $\dfrac{2m+3n}{3m+5n}=\dfrac{2p+3q}{3p+5q}$.

(6) If $\dfrac{a}{b}=\dfrac{c}{d}$, then $\dfrac{a^n+b^n}{a^n-b^n}=\dfrac{c^n+d^n}{c^n-d^n}$.

(7) $$\frac{\frac{1}{2}(x^{\frac{1}{2}}+3x^{-\frac{1}{2}})(x^{\frac{1}{2}}-2x^{-\frac{1}{2}})-\frac{1}{3}(x^{\frac{1}{2}}+2x^{-\frac{1}{2}})(x^{\frac{1}{2}}-3x^{-\frac{1}{2}})}{1-x^{-1}}$$
$$=\frac{x+6}{6}.$$

IX. SIMPLE EQUATIONS OF ONE UNKNOWN QUANTITY.

1. Solve the equations

(1) $2(x-3)+6(x-2)=54$.

(2) $5-3(4-x)+4(3-2x)=0$.

(3) $2ax+b=3cx+4a$.

(4) $2a(3a-x)-3x(2b-a)=ab$.

(5) $(2-x)(a-5)=4+3ax$.

(6) $\dfrac{3x}{4}+\dfrac{7x}{15}+\dfrac{11x}{6}-366=0$.

(7) $\dfrac{3x}{4}-\dfrac{2x}{7}=3\frac{2}{21}$. (8) $\dfrac{x}{a}+\dfrac{x}{b}=c$.

(9) $\frac{1}{x}+\frac{1}{2x}-\frac{1}{3x}=\frac{7}{3}$ (10) $\frac{a}{bx}+\frac{b}{ax}=a^2+b^2$.

(11) $\frac{x}{a}-1-\frac{dx}{c}+3ab=0$.

(12) $\frac{2x-5}{3}+x=\frac{3x-2}{5}+3$.

(13) $\frac{x}{2}-\frac{2x-11}{3}-\frac{x+3}{4}=0$.

(14) $\frac{x-1}{2}+\frac{2x-1}{7}=\frac{3x-2}{5}+\frac{x}{8}$.

(15) $\frac{x}{4}-\frac{x+4}{6}=\frac{4}{3}+\frac{24-x}{12}$.

(16) $\frac{x-3}{3}-\frac{x-4}{4}+\frac{x-6}{6}-\frac{x+2}{7}=0$.

(17) $x-\frac{x-2}{3}=\frac{x+23}{4}-\frac{10+x}{5}$.

(18) $5x-\frac{2x-1}{3}+1=3x+\frac{x+2}{2}+7$.

(19) $\frac{12-4x}{10}-\frac{2x+5}{5}=3+\frac{7x+60}{2}-50$.

(20) $\frac{8}{7}x+\frac{1}{6}(x-1)=x-4$.

(21) $\frac{5}{7}(x-9)+\frac{7}{9}(x-5)=\frac{8}{5}(x-7)-1\frac{2}{3}$.

(22) $3x-\frac{1}{2}(x-1\frac{1}{2})=9-\frac{1}{4}(5x-7)$.

(23) $\frac{1}{6}x-1\frac{2}{3}=8\frac{3}{5}+2(\frac{3}{5}x-1)-\frac{1}{3}(x+8)$.

(24) $\frac{1}{15}(2x-1)-\frac{1}{16}(3x-2)=\frac{1}{18}(x-12)-\frac{1}{24}(x+1)$.

(25) $\frac{1}{8}(x-a)-\frac{1}{24}\{m-(a-x)\}=\frac{3}{16}(m+x)-\frac{1}{80}(15a+16m)$.

(26) $\frac{3x^2-2x+1}{5} = \frac{(7x-2)(3x-6)}{35} + \frac{9}{10}$.

(27) $\frac{3x}{2} - \frac{1-x}{3x} = 2\left(x - \frac{x}{4}\right)$.

(28) $(a+x)(b+x) - a(b+c) = \frac{a^2c}{b} + x^2$.

(29) $\frac{2x}{a-2b} - 3 = \frac{x}{2a-b}$.

(30) $\frac{x+c}{x-c} = \frac{a+b}{a-b}$.

(31) $\frac{6x-3}{3x-8} - \frac{x-1}{x-4} - 1 = 0$.

(32) $\frac{p-q}{qx+r} = \frac{p+q}{px-r}$.

(33) $\frac{3-4x}{3(3-x)} + \frac{1}{2(1-x)} = 1\frac{1}{3}$.

(34) $\frac{2x+a}{3(x-a)} + \frac{3x-a}{2(x+a)} = 2\frac{1}{6}$.

(35) $\frac{1}{a(d-x)} + \frac{1}{b(d-x)} + \frac{1}{c(d-x)} = \frac{1}{e^2}$.

(36) $\frac{7x+1}{x-1} = \frac{35}{9}\frac{x+4}{x+2} + 3\frac{1}{9}$.

(37) $\frac{8x+5}{14} + \frac{7x-3}{6x+2} = \frac{4x+6}{7}$.

(38) $\frac{2x+1}{3} - \frac{4x+2}{3x-9} = \frac{2x-5}{3}$.

(39) $\frac{8}{x+3} - \frac{9}{2x+6} = \frac{15}{7x+2}$.

(40) $\frac{x}{2} - \frac{x-1}{3} + \frac{x+3}{1\frac{1}{2}} = \frac{4x-7}{2}$.

(41) $\frac{17-3x}{5} - \frac{2x+1}{1\frac{1}{2}} = \frac{1-4x}{3}$

(42) $\frac{x+2}{10} - \frac{x-\frac{1}{2}}{5} = \frac{5+2x}{2}$

(43) $$\frac{\frac{x}{5}+\frac{1}{2}}{3}-\frac{x-\frac{x}{2}}{2}+\frac{x}{5}=0.$$

(44) $$\frac{x}{2}-\frac{\frac{2x-3}{3}-\frac{3x-1}{4}}{\frac{x-1}{4}}=\frac{3}{2}\,\frac{x^2+2}{3x-2}.$$

(45) $$1-\frac{a}{a+x+\frac{x^2}{a-x}}=\frac{b}{a}.$$

(46) $$\frac{\frac{x}{a}-\frac{x}{b}}{\frac{b}{c}+\frac{c}{b}}-\frac{\frac{b}{a}-\frac{a}{b}}{\frac{c}{a}+\frac{a}{c}}=\frac{1}{c^3}.$$

(47) $$\frac{1}{x+\frac{1}{a}}-\frac{1}{a+\frac{1}{x}}=\frac{\frac{a^2}{x}-x}{1+\frac{1}{ax}}.$$

2. Prove the rule for transposing quantities in equations.

3. Explain the principles of the clearing of fractions.

X. PROBLEMS PRODUCING EQUATIONS OF ONE UNKNOWN QUANTITY.

1. Find that number to which if its seventh part be added, the sum will be greater by 11 than $\frac{3}{4}$ the original number.

2. There is a certain fraction, the denominator of which is greater than the numerator by 2; but if 1 be *taken from each, then* the denominator is twice as great *as the numerator:* find the fraction.

3. The sum of two numbers is 60, and their ratio is that of 2 to 3: what are they?

4. In what number of two digits is the second digit double of the first, and the sum of the two less than half the number by 12?

5. Divide 64 into two parts which shall be to each other as 7 to 9.

6. Divide a line 12 inches long into two parts, so that the greater shall be five times as great as the less.

7. Divide a given quantity p into two parts, so that the sum of their quotients by 5 and 4 respectively may be equal to q. What must be the value of q, that these two quotients may be equal?

8. What sum is that which is as much greater than 1*l.* as its half is less than 1*l.*?

9. The value of 50 coins, consisting of half-guineas and half-crowns, is 16*l.* 5*s.*: how many are there of each?

10. A man pays a bill of 15*s.* 2*d.* with 26 coins, some of them half-crowns and the rest fourpenny pieces: how many are there of each?

11. A and B jointly have a fortune of 4900*l.*; A invests the sixth part of his property in business, and B the fifth part, and each has the same remaining: how much had each?

12. A and B have together 8*s.*, A and C have 10*s.*, B and C have 12*s.*: what have they each?

13. A farmer bought 4 fields: for the second he gave half as much again as for the first; for the third half as much again as he gave for the first and second together; and for the fourth as much as he had given for the other

three; and he paid 7500*l.* in all: how much did he give for each field?

14. A man distributes a certain sum among his 4 children. He gives the third twice as much as the fourth, and 1*l.* besides; the second twice as much as the third, and 1*l.* besides; the first twice as much as the second, and 1*l.* besides. Having 5*l.* remaining, he gives it to the first, and thereby makes his share double that of the third and second together. Required the whole amount, and each share.

15. Out of a cask of wine, one-third of which had leaked away, 21 gallons were drawn; and then, being gauged, it appeared to be half full: how much did it hold?

16. Two-thirds of a college lecture having gone into the schools, and one man being 'æger,' the lecture is reduced to 7 men: how many men were there in at first?

17. A playing with B first loses $\frac{1}{3}$ of his money, then $\frac{1}{5}$ of what remains, and then $\frac{1}{4}$ of what he still has; and finds he has 15*s.* left: how much had he at first?

18. A person bought 16 yards of cloth, and if he had bought one yard less for the same money, each yard would have cost him 1*s.* more: how much did he give for it per yard?

19. After a ball, where one-third of the dances were quadrilles, a young lady found she had danced other dances five times as often as quadrilles, and had missed only 3 dances altogether: supposing these 3 to have been quadrilles, how many times did she dance?

20. A is now twice as old as B, but 10 years ago *he was three times* as old: find the present ages of A *and B.*

21. Nine years ago A was three times as old as B, but now he is only twice as old: required the respective ages of A and B.

22. A square floor would contain 17 square yards more than it does if each side were 1 yard longer: find its area.

23. Two trains, travelling one at 26 and the other at 30 miles an hour, start from London and Bristol respectively, these places being 120 miles apart: how long will it be before the trains meet?

24. A person walking at the rate of $4\frac{3}{4}$ miles an hour starts $1\frac{1}{2}$ hour after another person who walks only 4 miles an hour: when and where will he overtake him?

25. If A can do a piece of work in 8 days, which A and B together can do in 6 days, how long would B take to do it alone?

26. A and B can do a piece of work in 8 days, working together; A working alone can do it in 12 days: in how many days could B do it, working alone?

27. A and B have to dig a field, of which each takes a half; at the end of 5 days A has completed his half, but B has not; A and B then work together, and finish the field in one day: how long would B have taken to dig it alone?

28. A and B can do a piece of work in p and q days respectively: in how many days will they do it together? and how long would it take B to accomplish what A can do in q days?

29. A man, woman, and child could reap a field in 30 hours, the man doing half as much again as the woman, and the woman two-thirds as much again as the child: how many hours would they each take to do it *separately?*

30. A man provided a basket of penny oranges for a school feast; he gave $\frac{1}{3}$ of them to the teachers, $\frac{5}{6}$ of the remainder to the children, and had half a crown's worth of oranges left: how many were there in the basket?

31. The proceeds of a concert, for which twice as many tickets were sold at 3*s.* 6*d.* as at 5*s.*, after deducting 102*l.* 12*s.* to defray the expenses, were found to average exactly 1*s.* on each person who attended it: how many tickets were sold at each price?

32. In a cricket match A made 35 runs; C made half as many as B, D one-third as many as B, and B's score was just as much below A's as C's was above D's: find the scores of B, C, and D.

33. A boat which at the beginning of the races had twice as many above it as below it, gains four places, and leaves off with twice as many below it as above it: how many boats were entered?

34. A workman was employed for 60 days, on condition that for every day he worked he should receive 15 pence, and for every day he was idle he should forfeit 5 pence; at the end of the time he had 20 shillings to receive: required the number of days he worked.

35. Twenty-four persons agreed to subscribe the cost of a new boat; but 4 of the subscribers proving defaulters, those who remained had to pay each 10*s.* more than their due share: what was the cost of the boat?

XI. SIMULTANEOUS SIMPLE EQUATIONS.

1. Solve the equations

(1) $\left.\begin{matrix} 2x+3y=22 \\ 3x+4y=31 \end{matrix}\right\}$ (2) $\left.\begin{matrix} 2x+3y=17 \\ 5x-2y=14 \end{matrix}\right\}$

(3) $\left.\begin{matrix} 5x=43-7y \\ 11x+9y=69 \end{matrix}\right\}$ (4) $\left.\begin{matrix} x+ay=b \\ ax+by=c \end{matrix}\right\}$

(5) $\left.\begin{matrix} ax+by=c \\ mx+ny=d \end{matrix}\right\}$ (6) $\left.\begin{matrix} mx+ny=1 \\ nx-my=0 \end{matrix}\right\}$

(7) $\left.\begin{matrix} \frac{x}{3}+\frac{y}{5}=8 \\ \frac{x}{9}+\frac{y}{10}=1 \end{matrix}\right\}$ (8) $\left.\begin{matrix} \frac{x}{9}-\frac{y}{8}=1 \\ \frac{x}{6}+\frac{y}{4}=12 \end{matrix}\right\}$

(9) $\left.\begin{matrix} \frac{x}{3}+\frac{y}{2}=\frac{7}{6} \\ \frac{3x}{7}-\frac{2y}{3}=\frac{4}{21} \end{matrix}\right\}$ (10) $\left.\begin{matrix} \frac{x}{9}+\frac{y}{8}=43 \\ \frac{x}{8}+\frac{y}{9}=42 \end{matrix}\right\}$

(11) $\left.\begin{matrix} \frac{5x}{8}+y=1 \\ x+\frac{5y}{8}=5\frac{1}{2} \end{matrix}\right\}$ (12) $\left.\begin{matrix} \frac{2x}{5}+\frac{y}{6}=5 \\ \frac{5x}{2}-4y=1 \end{matrix}\right\}$

(13) $\left.\begin{matrix} \frac{x}{m}+\frac{y}{n}=k \\ \frac{x}{n}+\frac{y}{m}=l \end{matrix}\right\}$ (14) $\left.\begin{matrix} \frac{x}{a}+\frac{y}{b}=1-\frac{x}{c} \\ \frac{y}{a}+\frac{x}{b}=1-\frac{y}{c} \end{matrix}\right\}$

(15) $\left.\begin{matrix} \frac{x}{2a}+\frac{y}{3b}=1-\frac{x}{4c} \\ \frac{y}{3a}+\frac{x}{2b}=1+\frac{y}{4c} \end{matrix}\right\}$

(16) $\left.\begin{matrix} \frac{ax}{c}-\frac{by}{a}=0 \\ \frac{x}{b}+\frac{y}{c}=1 \end{matrix}\right\}$ (17) $\left.\begin{matrix} 2x-\frac{y-3}{5}=4 \\ 3y+\frac{x-2}{3}=9 \end{matrix}\right\}$

(18) $$\left.\begin{aligned} \frac{x-1}{3}+\frac{y-1}{2}&=\frac{2y}{3}-\frac{1}{6} \\ \frac{2x-4}{4}-\frac{2y+1}{5}&=\frac{y-3}{2} \end{aligned}\right\}$$

(19) $$\left.\begin{aligned} 2x-\frac{y+3}{4}&=7+\frac{3y-2x}{5} \\ 4y+\frac{x-2}{3}&=26\tfrac{1}{2}-\frac{2y+1}{2} \end{aligned}\right\}$$

(20) $$\left.\begin{aligned} x-\tfrac{1}{7}(y-2)&=5 \\ 4y-\tfrac{1}{3}(x+10)&=3 \end{aligned}\right\}$$

(21) $$\left.\begin{aligned} \tfrac{1}{7}(8x-5)+\tfrac{1}{3}(y+4)&=0 \\ x+\tfrac{1}{4}&=y+1 \end{aligned}\right\}$$

(22) $$\left.\begin{aligned} \tfrac{2}{3}(x+y)+2&=12 \\ \tfrac{2}{7}(x-y)+6&=8 \end{aligned}\right\}$$

(23) $$\left.\begin{aligned} \frac{2}{x}+\frac{3}{y}&=0 \\ \frac{x}{3}+\frac{y}{2}&=1\tfrac{2}{3} \end{aligned}\right\}$$

(24) $$\left.\begin{aligned} \frac{m}{x}+\frac{n}{y}&=a \\ \frac{n}{x}+\frac{m}{y}&=b \end{aligned}\right\}$$

(25) $$\left.\begin{aligned} \frac{11}{x}+\frac{12}{y}&=58 \\ \frac{12}{x}+\frac{11}{y}&=57 \end{aligned}\right\}$$

(26) $$\frac{x}{y}=\frac{a-x}{y-2b}=\frac{a}{b}.$$

2. Solve the equations

(1) $$\left.\begin{aligned} x+2y+3z&=10 \\ 2x+3y+4z&=16 \\ 4x+4y+5z&=25 \end{aligned}\right\}$$

(2) $$\left.\begin{aligned} 5x+2y-z&=6 \\ 2z-x-y&=6 \\ y+z-2x&=6 \end{aligned}\right\}$$

(3) $$\left.\begin{aligned} 2x-4y+9z&=28 \\ 7x+3y-5z&=3 \\ 9x+10y-11z&=4 \end{aligned}\right\}$$

(4) $$\left.\begin{aligned} x+y&=16 \\ 2z-2x&=8 \\ \frac{2y+3z}{17}&=3 \end{aligned}\right\}$$

(5) $\left.\begin{array}{l} \frac{1}{2}x+\frac{1}{3}y=12-\frac{1}{6}z \\ \frac{1}{2}y+\frac{1}{3}z=8+\frac{1}{6}x \\ \frac{1}{2}x+\frac{1}{3}z=10 \end{array}\right\}$ (6) $\left.\begin{array}{l} \frac{1}{x}+\frac{2}{y}=7 \\ \frac{2}{y}+\frac{3}{z}=21 \\ \frac{3}{z}+\frac{4}{x}=19 \end{array}\right\}$

(7) $\left.\begin{array}{l} \frac{2}{x}-\frac{5}{3y}+\frac{1}{z}=3\frac{4}{27} \\ \frac{1}{4x}+\frac{1}{y}+\frac{2}{z}=6\frac{11}{72} \\ \frac{5}{6x}-\frac{1}{y}+\frac{4}{z}=12\frac{1}{36} \end{array}\right\}$ (8) $\left.\begin{array}{l} \frac{xy}{x+y}=1 \\ \frac{xz}{x+z}=2 \\ \frac{yz}{y+z}=3 \end{array}\right\}$

XII. PROBLEMS PRODUCING SIMULTANEOUS EQUATIONS.

1. Find two numbers whose sum shall be triple of their difference and also less than 100 by the smaller of the two.

2. There is a certain fraction which, if 1 be added to its numerator, becomes $\frac{1}{3}$; but if 1 be added to its denominator, it becomes $\frac{1}{4}$: what is the fraction?

3. The sum of the two digits of a certain number is six times their difference, and the number itself exceeds six times their sum by 3: find it.

4. If the digits in a certain number consisting of two figures be inverted, the sum of the two numbers will be 121, and the difference between them will be 9: find the number.

5. If a father was four times as old as his son seven years ago, and will be twice as old as his son in seven years more, what is the present age of each?

6. A man sold 8 horses and 10 cows for 350*l.*, and 10 horses and 8 cows for 370*l.*: what was the price of each?

7. A farmer sells 19 horses and 7 cows for 547*l.*, and 6 horses and 31 cows for the same sum: what was the price of each?

8. From the termini of a railway a miles long two trains are despatched, one b minutes later than the other; at c minutes after the first train has started they meet half-way: if the speeds of the trains be interchanged, at what point will they meet?

9. An eleven at cricket scored 58; two scored nothing, seven scored 30 between them, and the difference between the two principal scores was 10: find what each of these two scores amounted to.

10. What fraction is that, to the numerator of which if 1 be added, its value will be $\frac{1}{2}$; but if 1 be added to the denominator, its value will be $\frac{1}{3}$?

11. A man bought 3 horses and 5 cows, and gave the same sum for the 3 horses as for the 5 cows; if he had bought 4 horses and 10 cows, his outlay would have been 120*l.* more: find what his outlay was.

12. Two numbers consist of the same two digits, but in an inverse order; the sum of the numbers is 55, and their difference is 27: find them.

13. A number consists of three digits, the right-hand one being 0; if the left-hand and the middle digits be interchanged, the number will be diminished by 180; if the left-hand digit be halved, and the middle and right-*hand digits* be interchanged, the number will be dimi*nished by 454: find* the number.

14. A, B, C, and D have 290*l.* between them: A has twice as much as C, and B three times as much as D; also C and D together have 50*l.* less than A: find how much each has.

15. The difference between two numbers is one-fourth of their sum; and the half of the greater, diminished by 7, is equal to one-fifth of the less, increased by $2\frac{1}{2}$: find the numbers.

XIII. INVOLUTION.

1. Square

(1) $1-2x+3x^2-4x^3$. (2) $1+\frac{1}{2}x+\frac{1}{3}x^2+\frac{1}{4}x^3$.

(3) $x^{-1}-1+x$. (4) $a-bx+cx^2$.

(5) $\frac{x^2}{y^2}+\frac{y^2}{x^2}+\frac{x}{y}+\frac{y}{x}+1$. (6) $\frac{1}{1+\frac{x}{a}}-\frac{1}{1-\frac{x}{a}}+\frac{2}{\frac{x}{a}+\frac{a}{x}}$.

2. Cube

(1) $2x+3$. (2) $2a-5b$. (3) $\frac{x}{2}-\frac{y}{3}$.

(4) $\frac{p^2}{2}-\frac{2q^2}{3}$. (5) $ax^{\frac{1}{2}}-by^{\frac{1}{2}}$. (6) e^x-e^{-x}.

(7) $\frac{x^2}{y^2}-2+\frac{y^2}{x^2}$. (8) $e^x-e^{-x}+1$.

3. Expand

(1) $(2x^2yz^3)^5$. (2) $(-7a^3b^4c^5)^3$. (3) $(-3ab^4c^2)^5$.

(4) $\left(\frac{a}{b}+\frac{b}{a}\right)^3$. (5) $\left(\frac{2x}{y}-\frac{y}{2x}\right)^3$. (6) $\left(\frac{x^2}{y^2}-\frac{y}{x}\right)^3$.

(7) $\left(2a-\frac{3}{x}\right)^3$. (8) $\left(\frac{a}{2}+\frac{2b}{3}\right)^3$ (9) $\left(\frac{a}{3}-\frac{3b}{ac}\right)$.

(10) $(2x^{\frac{1}{2}}y^{\frac{1}{3}}-\frac{1}{2}x^{\frac{1}{3}}y^{\frac{1}{2}})^6$. (11) $(a-3b)^4$.

(12) $\left(l-m-\frac{1}{lm}\right)^2$. (13) $\left(x-\frac{1}{x}-1\right)^3$.

4. Give the rule for forming the square of a multinomial.

XIV. EVOLUTION.

1. Find the square root of

(1) $25a^4b^2c^2+10a^2bc^5+c^8$. (2) $\frac{x^2}{25}+25y^2-2xy$.

(3) $4x^6-2+\frac{1}{4x^6}$. (4) $\frac{a^2}{b^2}+\frac{b^2}{a^2}-2$.

(5) $a^{2m}-4a^{m+n}+4a^{2n}$.

(6) $16a^{2m}-24+9a^{-2m}$. (7) $4a^{\frac{2}{3}}+4a^{\frac{1}{3}}b^{\frac{1}{3}}+b$.

(8) $x^4-8x^3-26x^2+168x+441$.

(9) $4x^4-12x^3+25x^2-24x+16$.

(10) $1+4x+10x^2+12x^3+9x^4$.

(11) $9x^4-12x^3y+34x^2y^2-20xy^3+25y^4$.

(12) $10x^4-10x^3-12x^5+5x^2+9x^6-2x+1$.

(13) $x^6-2px^5+p^2x^4-2rx^3+2prx^2+r^2$.

(14) $x^6+10ax^5+35a^2x^4+52a^3x^3+35a^4x^2+10a^5x+a^6$.

(15) $9x^8-12ax^7+4a^2x^6+6a^3x^5-4a^4x^4+a^6x^2$.

(16) $x^4+2x^3-x+\frac{1}{4}$.

17) $x^4-2x^3+\frac{3x^2}{2}-\frac{x}{2}+\frac{1}{16}$.

18) $x^4-ax^3+\frac{a^2x^2}{4}+\frac{4}{a^2x^2}+\frac{4x}{a}-2$.

(19) $9x^2-6x-1+\frac{2}{3x}+\frac{1}{9x^2}$.

(20) $x^4-x^3+\frac{x^2}{4}+4x-2+\frac{4}{x^2}$.

(21) $\frac{a^2}{b^2}+\frac{b^2}{a^2}-\frac{a}{b}+\frac{b}{a}-\frac{7}{4}$.

(22) $4a^2+c^2+(4a+3b)(3b-2c)+4ac$.

(23) $a^2b^2+2ab(bc-ca)+b^2c^2+a^2c^2-2abc^2$.

(24) $9(a^2-1)^2-12(a^2-1)a+4a^2$.

(25) $a^4+b^{-4}+4(a^3b^{-1}+a^{-1}b^3)+6a^2b^{-2}$.

(26) $a^6+b^{-6}+4a^5b^{-1}+2b^{-5}a+2a^4b^{-2}-3b^{-4}a^2$
$-6a^3b^{-3}$.

(27) $\frac{9a^2}{16x^2}+\frac{4}{9}\left\{1+\frac{x}{a}\right\}-\left\{\frac{a}{x}+\frac{1}{2}\right\}+\frac{x^2}{9a^2}$.

(28) $\frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}+2\left(\frac{y}{x}+\frac{z}{y}+\frac{x}{z}\right)$.

(29) $\frac{x^2}{y^2}\left(\frac{x^2}{4y^2}+1\right)+\frac{4y^2}{x^2}\left(\frac{y^2}{x^2}+1\right)+3$.

(30) 20449, 2·0449. (31) 19321, 1·9321, 19·321.

(32) 6290·0761, ·6290761, 6290761·1.

2. Find the cube root of

(1) $a^3+3a^2b+3ab^2+b^3$.

(2) $a^3x^3-12a^2bx^3+48ab^2x^3-64b^3x^3$.

(3) $8a^3-36a^2b+54ab^2-27b^3$.

(4) $a^{-3}+6a^{-2}+12a^{-1}+8$

(5) $x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)$.

(6) $\frac{27}{8}-\frac{27}{4}a^{-\frac{1}{2}}+\frac{9}{2}a^{-1}-a^{-\frac{3}{2}}$.

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XV. SURDS.

1. Find the value of

$$\sqrt[4]{\frac{81a^4b^8c^{12}}{256x^{16}}} \text{ and } \sqrt[6]{\frac{64x^{12}y^6}{729z^{18}}}.$$

2. Write in simpler forms

(1) $(\sqrt{x})^{\frac{2}{3}}$. (2) $\sqrt[n]{(ax)^n}$. (3) $5ax\sqrt{\frac{a}{5x}}$.

(4) $\sqrt[6]{2a^3-3a^3}$.

3. Multiply together $2y^{\frac{1}{2}}\sqrt{2x^3}$, $\sqrt{6xy}$, and $\frac{x^{\frac{1}{2}}}{\sqrt{3y^{\frac{1}{2}}}}$

4. Simplify

(1) $\frac{1}{a-\sqrt{a^2-x^2}}-\frac{1}{a+\sqrt{a^2-x^2}}$.

(2) $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$.

5. Find product of

$$\sqrt{x^3+1+\frac{1}{\sqrt{x^3}}} \text{ and } \sqrt{x^3-1+\frac{1}{\sqrt{x^3}}}.$$

6. $\left(2a+\frac{1}{b}\right)\left(2b+\frac{1}{a}\right)=\left(2\sqrt{ab}+\frac{1}{\sqrt{ab}}\right)^2$.

7. Find the value of

$$\frac{\sqrt{2mn+y}+\sqrt{2mn-y}}{\sqrt{2mn+y}-\sqrt{2mn-y}} \text{ when } y=\frac{4mnp}{p^2+1}.$$

8. *If* $x=\frac{\sqrt{2}+1}{\sqrt{2}-1}$, and $y=\frac{\sqrt{2}-1}{\sqrt{2}+1}$,

find the value of x^2-xy+y^2.

9. Simplify

(1) $\sqrt{12}+\frac{1}{10}\sqrt{75}+6\sqrt{\frac{1}{18}}$.

(2) $\sqrt{24}\times\sqrt{\frac{8}{3}}\times\sqrt{11\frac{1}{16}}$.

(3) $(2\sqrt{3}+\sqrt{2})^2-(2\sqrt{3}-\sqrt{2})^2$

(4) $\dfrac{1}{2\sqrt{7}-3\sqrt{2}}-\dfrac{1}{2\sqrt{7}+3\sqrt{2}}$.

(5) $\left\{\dfrac{\sqrt{3}-1}{\sqrt{3}+1}\right\}^{-\frac{1}{2}}-\left\{\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right\}^{-\frac{1}{2}}$.

10. Which is the greater, $\sqrt{14}+\sqrt{7}$ or $\sqrt{19}+\sqrt{2}$? Prove your answer.

11. Shew that a surd cannot equal the sum or difference of a rational quantity and a surd.

12. Find the square root of

(1) $13+4\sqrt{3}$. (2) $14-2\sqrt{33}$.

(3) $7\frac{1}{4}+3\sqrt{5}$. (4) $17+20\sqrt{-2}$

13. Solve the equations

(1) $a-x=\sqrt{2ax+x^2}$. (2) $a+x+\sqrt{a^2+x^2}=b$.

(3) $\sqrt{x+1}=\sqrt{x}+\frac{1}{2}$.

(4) $\sqrt{x+20}-\sqrt{3}=5-\sqrt{x-2}$.

(5) $\sqrt{x-4}=\dfrac{259-10x}{\sqrt{x}+4}$.

(6) $\dfrac{5x-9}{\sqrt{5x}+3}-1=\dfrac{\sqrt{5x}-3}{2}$.

(7) $\dfrac{ax-1}{\sqrt{ax}+1}=4+\dfrac{\sqrt{ax}-1}{2}$.

(8) $\sqrt{x}-\sqrt{2+x}=\sqrt{\dfrac{2}{x}}$.

(9) $\dfrac{\sqrt{x}+28}{\sqrt{x}+4}=\dfrac{\sqrt{x}+38}{\sqrt{x}+6}$.

XVI. QUADRATIC EQUATIONS OF ONE UNKNOWN QUANTITY.

1. Solve the equations

(1) $(x-1)(x-2)=5(x-3)+2$.

(2) $(x-1)^2+1=x$.

(3) $(x+10\frac{1}{3})^2=4(x+9\frac{1}{3})$.

(4) $\frac{1}{3}(x-1)=\frac{1}{2}(x+1)-\frac{1}{6}(x-1)^2$.

(5) $7x^2-11x=6$.

(6) $\dfrac{2x-3}{4}-\dfrac{x-1}{2x}=\frac{1}{7}$.

(7) $5x+\dfrac{4x+3}{3x-2}=\dfrac{10x^2-7}{2x-1}+9$.

(8) $\dfrac{9x^2+1}{3x-2}-3x=\dfrac{5x-1}{4x-3}+3$.

(9) $x-15\frac{3}{4}+\dfrac{5}{x-15\frac{3}{4}}=6$. (10) $\dfrac{15}{x+\frac{1}{4}}=\dfrac{15}{x}-\frac{1}{4}$.

(11) $x+9\frac{5}{8}+\dfrac{1}{\dfrac{x}{7}+\dfrac{11}{8}}=8$.

(12) $ax^2+b(2a+b)x-(a^3-b^3)=0$.

(13) $(x+a)(a^2-ax+b^2)=a^3+bx^2$.

(14) $\dfrac{x^2-1}{9}+\dfrac{1}{x^2}=1$.

(15) $x^4-4bcx^2-(b^2-c^2)^2=0$.

(16) $\left(x-\dfrac{1}{x}\right)^2-\dfrac{25}{6}\left(x-\dfrac{1}{x}\right)+4=0$.

2. Solve the equation $x^2-px+q=0$; and shew that the sum of the roots is equal to p, and their product is equal to q.

3. If α, β are the roots of the equation $ax^2+bx+c=0$, prove that $ax^2+bx+c=a(x-\alpha)(x-\beta)$.

4. Find the relation between p and q when the roots of the equation $x^2-px+q=0$ are equal.

5. What is the relation between a and c when the product of the roots of the equation $ax^2+bx+c=0$ is equal to unity?

6. Shew that the roots of a quadratic equation are either real and equal, real and unequal, or both impossible.

XVII. QUADRATIC EQUATIONS OF TWO UNKNOWN QUANTITIES.

1. Solve the equations

(1) $\left.\begin{aligned} x^2+y^2&=25\\ x+y&=7\end{aligned}\right\}$ (2) $\left.\begin{aligned} x^2+y^2&=29\\ x-y&=3\end{aligned}\right\}$

(3) $\left.\begin{aligned} x+y&=8\\ x^2+2y^2&=43\end{aligned}\right\}$ (4) $\left.\begin{aligned} 3x+2y&=13\\ xy&=6\end{aligned}\right\}$

(5) $\left.\begin{aligned} 3x-y&=3\\ 3x^2+2xy-y^2&=9\end{aligned}\right\}$ (6) $\left.\begin{aligned} x^2-xy&=10\\ xy-y^2&=6\end{aligned}\right\}$

(7) $\left.\begin{aligned} xy-3x^2&=5\\ y^2-3xy&=19\end{aligned}\right\}$ (8) $\left.\begin{aligned} x^2+y^2&=29\\ \tfrac{1}{4}(2x-y)&=y-\tfrac{3}{4}(y-2)\end{aligned}\right\}$

(9) $x-\dfrac{1}{y}=y-\dfrac{1}{x}=\frac{3}{2}$.

(10) $x+y+2xy=3(x+y)-xy=7$.

(11) $2(x-y)+xy=3xy-(x-y)=7$.

(12) $\left.\begin{aligned} x^2-xy+y^2&=\tfrac{13}{25}\\ x^2+y^2&=1\end{aligned}\right\}$ (13) $\left.\begin{aligned} y^2-xy&=15\\ x^2+xy&=14\end{aligned}\right\}$

(14) $\left.\begin{aligned} x^2+xy+y^2&=1\tfrac{3}{4}\\ 2x^2-3xy+2y^2&=1\tfrac{1}{4}\end{aligned}\right\}$ (15) $\left.\begin{aligned} x^2-3xy+2y^2&=3\\ xy+y^2&=6\end{aligned}\right\}$

(16) $\left.\begin{aligned} x^2-y^2&=7\\ xy&=12\end{aligned}\right\}$ (17) $\left.\begin{aligned} \frac{x}{a}+\frac{y}{b}&=1\\ \frac{b}{x}+\frac{a}{y}&=1\end{aligned}\right\}$

XVIII. PROBLEMS PRODUCING QUADRATIC EQUATIONS.

1. Find two numbers such that their difference is 18, and the square of the one is 9 times the other.

2. The sides of a rectangle are 16 and 24 feet respectively: what breadth of border must be taken off all round, that the remaining area may be 240 square feet?

3. A person bought 2 pieces of cloth, one 6 yards longer than the other, for 20*l.* 10*s.*; and each cost as many shillings per yard as there were yards in its length: find the length of each.

4. A man walking $\frac{1}{3}$ of a mile above his ordinary rate gains $\frac{3}{4}$ of an hour in 39 miles: at what rate does he ordinarily walk?

5. A certain number of persons, going for an excursion, took two friends with them; the whole expenses were 32*l.* 6*s.*, and each share was 4*s.* more than it would have been without the friends: of how many persons did the party consist?

6. A person bought a certain quantity of tea for 8*l.* 5*s.*; and if he had given sixpence a pound less he might have had 3lbs. more for the same money. How much did he buy? and what did he give for it per lb.?

7. *A person* out walking has 18 miles to go, and finds *that at the rate* at which he is going he will be half an

hour late, but if he quickens his pace by $\frac{1}{2}$ a mile an hour he will arrive just at the proper time: at what pace is he going?

8. A certain number, of two digits, is equal to three times the product of its digits; and if 18 be added to the number its digits will be transposed: what is the number?

9. The area of a field is $7\frac{1}{2}$ acres, and the distance between the opposite corners is 275 yards: find the lengths of the sides.

10. Find two numbers whose sum is 7, and the sum of their squares exceeds their product by 19.

XIX. RATIO AND PROPORTION.

1. Which is the greater ratio, $4 : \sqrt{17}$ or $5 : \sqrt{26}$?

2. Find a number which, when increased by 4, has the same ratio to 61 which, if diminished by 4, it would have to 59.

3. Find the fourth proportional to 6, 9, 12; the third proportional to 8, 12; and the mean proportional between 12, 27.

4. Find three numbers in the proportion of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$, such that the sum of their squares is 724.

5. If $a : b :: c : d$, prove that

(1) $ma+nc : mb+nd :: ma-nc : mb-nd$.

(2) $ma^2-nb^2 : pa^2+qb^2 :: mc^2-nd^2 : pc^2+qd^2$.

(3) $a : b :: (ma^2+nc^2)^{\frac{1}{2}} : (mb^2+nd^2)^{\frac{1}{2}}$.

(4) $(a+b)^{\frac{1}{2}} : b^{\frac{1}{2}} :: (c+d)^{\frac{1}{2}} : d^{\frac{1}{2}}$.

(5) $\frac{a}{p}+\frac{b}{q} : a :: \frac{c}{p}+\frac{d}{q} : c$.

(6) $\frac{b}{a}+\frac{a}{b} : \frac{ab}{a^2+b^2} :: \frac{d}{c}+\frac{c}{d} : \frac{cd}{c^2+d^2}$.

6. If $a : b :: c : d :: e : f$, prove that

(1) $a : b :: \sqrt{ce} : \sqrt{df}$.

(2) $(ace)^{\frac{1}{3}} : (bdf)^{\frac{1}{3}} :: (ma^2+nc^2+pe^2)^{\frac{1}{2}} : (mb^2+nd^2+pf^2)^{\frac{1}{2}}$.

7. If $a : b :: b : c$, prove that

(1) $a+b : b+c :: (ab)^{\frac{1}{2}} : (bc)^{\frac{1}{2}}$.

(2) $\frac{a^n+b^n}{b^n+c^n}=\frac{a^n-b^n}{b^n-c^n}=\left(\frac{a}{c}\right)^{\frac{n}{2}}$.

8. If $a : b :: b : c :: c : d$, prove that

(1) $a : b :: a^{\frac{1}{3}} : d^{\frac{1}{3}} :: (ab+ac+bc)^{\frac{1}{2}} : (bc+bd+cd)^{\frac{1}{2}}$.

(2) $ac : bd :: a^2+b^2+c^2 : b^2+c^2+d^2$.

9. If $a+b : b :: b+c : c :: c+a : a$, shew that $a=b=c$.

10. If $a+c : b+c :: a^2 : b^2$, then either $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$, or $a=b$.

11. Find the ratios of $x : y : z$ from the equations $ax+by+cz=0$, $lx+my+nz=0$.

XX. DEFINITIONS OF TERMS, &c.

1. Explain the terms coefficient, power, root, index, factor, term; and prove that the product of two negative quantities is positive.

2. Explain the terms multiple, two dimensions, reciprocal of a quantity.

3. What is an impossible quantity, and what is an irrational quantity? Define Involution, Evolution, Exponent.

4. Explain what you mean by simple factors, binomial expressions, homogeneous quantities, similar surds.

EXAMINATION PAPERS.[1]

ALGEBRAICAL.

I.

1. Simplify

$$2-3x-(4-6x)-\{7-(9-2x)\}.$$

2. Multiply $1+2x+3x^2$ by $1+4x+5x^2+6x^3$, and find the continued product of $x+y$, $x-y$, and $x^2+2xy+y^2$.

3. Divide $x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$ by $x+a$.

4. Reduce to its simplest form $\dfrac{a^2+ab+b^2}{a^2-ab+b^2}\times\dfrac{a^3+b^3}{a^3-b^3}$.

5. Find the G.C.M. of $x^3+6x^2+11x+6$ and $x^3+9x^2+27x+27$.

6. Simplify $\dfrac{3x^2+1}{x^2-1}-\dfrac{5x-2}{x+1}+\dfrac{2x+5}{x-1}$.

7. Extract the square root of

$$x^4+\frac{2x^3}{3}+\frac{10x^2}{9}+\frac{x}{3}+\frac{1}{4}.$$

8. Extract the cube root of

$$x^6+6x^5+21x^4+44x^3+63x^2+54x+27.$$

[1] Examination Papers I.–XXVIII. extend only to Simple Equations.

9. Solve the equations

(1) $\frac{x}{4}-\frac{x-5}{3}=\frac{x-2}{6}$.

(2) $\frac{x-3}{7}-\frac{\frac{x}{2}-3}{3}=\frac{\frac{x}{6}+2}{2}-\frac{x-6}{3}+\frac{x}{8}$.

(3) $ax-\frac{b^2}{a}x=a+b$.

(4) $\left.\begin{aligned}\frac{x}{4}+\frac{y}{2}&=8\\ \frac{x}{2}+\frac{y}{6}&=6\end{aligned}\right\}$

10. A is twice as old as B and 4 years older than C; the sum of the ages of A, B, and C is 96 years: find B's age.

II.

1. Reduce to their simplest forms

(1) $\{y^2-(1-y)\}\,x-\{x^2+(x-1)y\}$

(2) $1-\{1-(1-4x)\}+\{2x-(3-5x)\}-\{2-(-4+5x)\}$.

2. Multiply px^2+qx-r by $mx-n$, and collect coefficients of the powers of x in the product.

3. Multiply and divide a^m by a, by a^n, and by a^{m+1}; also a^2 by $a^{\frac{1}{2}}$, by a, and by $a^{\frac{3}{2}}$; and divide

$max^3+(mb-na)z^2-(mc+nb)z+nc$ by $mz-n$.

4. Extract the square root of $\frac{a^2}{9b^2}+\frac{9b^2}{a^2}-2$, and the cube root of $8x^3-84x^2y+294xy^2-343y^3$.

5. Simplify as much as possible

(1) $\frac{a^4-b^4}{a^5-a^3b^2}$. (2) $\frac{a^2+b^2-c^2+2ab}{a^2-b^2-c^2+2bc}$.

(3) $\dfrac{a+1}{b} \div \dfrac{1+\frac{1}{a}}{b}$. (4) $\dfrac{a}{1-a}+\dfrac{a}{1+a}+\dfrac{1}{1+a}$.

6. Solve the equations

(1) $\dfrac{3x-2}{3}+\dfrac{90x-9}{9}=5x+4\frac{1}{3}$.

(2) $ax+b^2=a^2+bx$. (3) $\sqrt{x+9}=1+\sqrt{x}$.

(4) $\left.\begin{array}{l}\dfrac{2x-y}{4}-\dfrac{3}{2}=\dfrac{3y}{4}-x-2 \\ \dfrac{x+y}{3}=2\frac{2}{3}\end{array}\right\}$

7. Find the G.C.M. of

$12a^2+13ab+3b^2$, and $6a^2+23ab+7b^2$,

and the L.C.M. of a^2-1, $(ab-b)c$, $(ac+c)b$.

8. Find the value of $\dfrac{x}{x-3}-\dfrac{x-3}{x}+\dfrac{x}{x+3}-\dfrac{x+3}{x}$;

and of $\left(p+\dfrac{pq}{p-q}\right)\times\left(p-\dfrac{pq}{p+q}\right)\times\dfrac{p^2-q^2}{p^2+q^2}$

9. A and B have each an annual income of 400*l.*; A spends every year 40*l.* more than B, and at the end of 4 years the amount of their joint savings is equal to one year's income of either: what does each spend annually

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III.

1. Find the value of

$a^6-3a^5b+6a^4b^2-7a^3b^3+6a^2b^4-3ab^5+b^6$,

when $a=1$, $b=2$.

2. Divide $x^2+y^2+z^2+2(xy+yz+xz)$ by $x+y+z$, and $x^{4a}+4y^{4b}$ by $x^a+2x^ay^b+2y^{2b}$.

3. Find the square root of

$$1-xy-\frac{15}{4}x^2y^2+2x^3y^3+4x^4y^4,$$

and the cube root of

$$1-6x+21x^2-44x^3+63x^4-54x^5+27x^6.$$

4. Find three numbers in the proportion of 1, 2, 3, the sum of whose cubes is 288.

5. Prove the rule for finding the G.C.M. of two quantities, and find the G.C.M. of

$$x^2-y^2+xz+yz \text{ and } x^2+y^2+xz-yz-2xy.$$

6. Find the L.C.M. of $a+b$, $b-c$, $c+a$, a^2-b^2, b^2+c^2, c^2-a^2.

7. Simplify

(1) $b^3\sqrt[3]{8a^6b^3}+4a\sqrt[3]{a^3b^{12}}-\sqrt[3]{125a^6b^{12}}$.

(2) $\dfrac{\frac{2}{3}(2-\frac{1}{2}a)-a}{2-\frac{1}{2}(2+2a)}-\dfrac{\frac{3}{4}(6a+4)+\frac{1}{2}a}{1+2a}+2$.

(3) $\left\{\dfrac{1}{3}+\dfrac{2a}{3(1-a)}\right\}\times\left\{\dfrac{3}{4}-\dfrac{3a}{2(1+a)}\right\}$.

8. Prove that if $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}$, then $\dfrac{a}{b}=\dfrac{xa+yc+ze}{xb+yd+zf}$.

9. Solve the equations

(1) $\dfrac{2\frac{1}{3}-\frac{1}{2}(x-2)}{\frac{1}{3}(x+1)-4\frac{1}{2}}=1$.

(2) $\dfrac{2x+a}{b}-\dfrac{x-b}{a}=\dfrac{3ax+(a-b)^2}{ab}$.

(3) $x+y=5(x-y)=10$.

(4) $\left.\begin{aligned} x+y+z&=6\\ x+y-z&=4\\ x-y+z&=2\end{aligned}\right\}$ (5) $\left.\begin{aligned} \frac{x}{2}-1&=\frac{y}{3}+1=\frac{z}{4}\\ x+y+z&=17\end{aligned}\right\}$

10. A traveller walks a certain distance at a certain rate: if he had gone half a mile an hour faster, he would have walked it in four-fifths of the time; if he had gone half a mile an hour slower, he would have been two hours and a half longer on the road: find the distance and the rate at which he travelled.

IV.

1. Simplify

(1) $\dfrac{b}{1-\dfrac{a}{a-b+\dfrac{b^2}{a+b}}}$. (2) $\dfrac{\frac{1}{a}+\frac{1}{2}\left(\frac{1}{b}+\frac{2}{3c}\right)}{\frac{1}{a}-\frac{1}{3}\left(\frac{1}{b}-\frac{3}{2c}\right)}$.

(3) $2(x-y)-2\left\{3(x-y)-(2x-3y-\dfrac{x-4}{2})\right\}$.

(4) $\dfrac{a^2x}{b+x}\times\dfrac{b^2x}{a-x}\div\dfrac{(ax+b)^2-(ax-b)^2}{ab-(a-b)x-x^2}$.

2. Divide

(1) $1-5x+10x^2-10x^3+5x^4-x^5$ by $1-2x+x^2$.

(2) $x^3-(p+a)x^2+(q+ap)x-aq$ by $x-a$.

3. Multiply

(1) $\dfrac{2x}{3}-\dfrac{1}{2}$ by $\dfrac{x^2}{2}-\dfrac{x}{3}+\dfrac{1}{4}$. (2) $x^{\frac{3}{2}}-4x^{\frac{1}{2}}+2$ by $x-x^{\frac{1}{2}}$.

(3) $\dfrac{1}{1+\frac{1}{x}}+\dfrac{1}{1-\frac{1}{x}}$ by $\dfrac{x^2+x+1}{2(x+1)+\frac{2}{x-1}}$.

(4) $a^2-a+1-a^{-1}$ by $a+1+a^{-1}+a^{-2}$.

4. Find the G.C.M. of

$$3x^3-3x^2y+xy^2-y^3 \text{ and } 4x^2-xy-3y^2,$$

and the L.C.M. of

$$9x^3+53x^2-9x-18 \text{ and } x^2+11x+30.$$

5. Find the square root of $x^4-4x^3+x^2+6x+\frac{9}{4}$, and the cube root of $a^3-a^2b+\frac{ab^2}{3}-\frac{b^3}{27}$.

6. If $\frac{a}{b}=\frac{c}{d}$, shew that $\frac{a+b}{c+d}=\frac{a-b}{c-d}$.

7. Find the value of

$$2(1+\sqrt{a})-\frac{\sqrt{x}-\sqrt{a}}{3}+\frac{x-a}{\sqrt{a}+\sqrt{x}}$$

when $x=16a$.

8. Solve the equations

(1) $x+a+b+c=\frac{x^2+a^2+b^2+c^2}{a+b-c+x}$.

(2) $$\frac{7x+\frac{13}{2}}{10}+\frac{11x-\frac{x-\frac{3}{2}}{2}}{12}=\frac{3x+1}{5}+\frac{x-\frac{3-8x}{2}}{22}.$$

(3) $$\left.\begin{array}{l} x+y-8=0 \\ \frac{x-y}{2}+\frac{2x-3}{3}+\frac{4}{3}=0 \end{array}\right\}$$

(4) $$\left.\begin{array}{l} 3x+4y-5z=32 \\ 4x-5y+3z=18 \\ 5x-3y-4z=2 \end{array}\right\}$$

(5) $$\left.\begin{array}{l} 3x=2(y+2)-\frac{1}{3} \\ 2y=3(z-x)+\frac{3}{2} \\ z=x+y-\frac{2}{3} \end{array}\right\}$$

9. A boat is rowed one mile in $5\frac{1}{2}$ minutes; another boat is rowed the same distance in $5\frac{3}{4}$ minutes: if they

start simultaneously from opposite ends of a 4-mile course, at what distance from either starting point will they meet?

V.

1. Find the value of $a(a+3c)-b^2+c(c-a)$, when $a=3$, $b=4$, $c=1$.

2. Find the product of $\frac{2x}{a}-\frac{3y}{b}$ and $\frac{4x^2}{a^2}+\frac{6xy}{ab}+\frac{9y^2}{b^2}$.

3. Prove that a^n-b^n is divisible by $a-b$.

4. Resolve into their elementary factors
$a^4x^4-\frac{81}{16}y^4$, $27bx^4-b^4xy^3$, $20ax^2+18a^2x-18a^3$.

5. Simplify

(1) $$\frac{\left(x+\frac{1}{x}\right)^2-2\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2}.$$

(2) $$\frac{a^2x^3-2a^3x^2+a^4x}{x^2-ax+a^2}\times\frac{x^4+a^2x^2+a^4}{(x-a)\ (2ax+3ay)}\div\frac{(ax^2-a^2x)\ (x^2+ax+a^2)}{2x+3y}.$$

6. Find the G.C.M. of

$7a^2-23ab+6b^2$ and $5a^3-18a^2b+11ab^2-6b^3$,

and L.C.M. of

x^2-4y^2 and $x^2+xy-6y^2$.

7. Extract the square root of

$$x^4-x^3+\frac{5x^2}{4}-\frac{x}{2}+\frac{1}{4}.$$

8. Solve the equations

(1) $(2+x)(8+x)-22=\frac{3}{2}+x^2$.

(2) $(a+x)(b+x)-a(b+c)=\frac{a^2c}{b}+x^2$.

(3) $\frac{7x+5}{3}-\left(2x-\frac{3x-7}{14}\right)=5$.

(4) $\left.\begin{aligned}\frac{x}{7}+7y&=99\\ \frac{y}{7}+7x&=51\end{aligned}\right\}$

(5) $\left.\begin{aligned}x+y+z&=0\\ ax+by+cz&=0\\ 2x+3y+z&=1\end{aligned}\right\}$

9. There are three rods whose lengths are proportioned as 1, 2, 3; their combined length $=42$ feet: find their respective lengths.

10. 36 octavo volumes exactly fill a box whose length and breadth are $1\frac{1}{2}$ foot and $11\frac{1}{3}$ inches respectively: the dimensions of each volume being 9 inches by $5\frac{2}{3}$ inches and by $1\frac{1}{2}$ inch, find the depth of the box.

VI.

1. Add $\frac{1}{2}a-\frac{1}{3}b+\frac{1}{8}c$, $\frac{1}{4}a-\frac{1}{8}b-\frac{1}{3}c$, and $\frac{1}{3}a+\frac{1}{4}b+\frac{1}{2}c$; and subtract $3x-\frac{x^2}{8}-(2x+3x^2)$ from $\frac{x^2}{2}-\frac{5x}{6}-\left(\frac{3x}{4}-x^2\right)$.

2. Divide $5x^4+\frac{7ax^3}{2}-\frac{107}{12}a^2x^2+\frac{5a^3x}{6}+\frac{7a^4}{6}$

by $2x^2-ax-\frac{a^2}{2}$.

3. Reduce to their simplest terms

(1) $a^2+ab+b^2+\frac{b^3}{a-b}$.

(2) $\dfrac{1}{1+x+\dfrac{1}{1+x+\dfrac{1}{1+x}}}$.

(3) $\dfrac{1}{4(1+x)}+\dfrac{1}{4(1-x)}+\dfrac{1}{2(1+x^2)}$.

4. Multiply $a^m+b^m+c^m$ by $a^n+b^n+c^n$, and $a^{\frac{3}{4}}+a^{\frac{1}{2}}b^{\frac{1}{2}}+a^{\frac{1}{4}}b+b^{\frac{3}{2}}$ by $a^{\frac{1}{4}}-b^{\frac{1}{2}}$.

5. Find the G.C.M. of

$20x^4+x^2-1$ and $25x^4+5x^3-x-1$,

and the L.C.M. of $x^2-9x+20$ and $x^2+6x-55$.

6. Resolve into their elementary factors

(1) $a^3-a^2x-ax^2+x^3$. (2) $x^3+3x^2y-4xy^2-12y^3$.

7. Find the square root of $36x^4-36x^3+17x^2-4x+\frac{4}{9}$.

8. Solve the equations

(1) $\dfrac{8x+5}{14}+\dfrac{7x-3}{6x+2}=\dfrac{16x+15}{28}+\dfrac{2\frac{1}{4}}{7}$.

(2) $\dfrac{2x+1}{3}-\dfrac{3x-2}{5}+\dfrac{5x-\frac{x+\frac{3}{2}}{2}}{6}=9\frac{7}{12}$.

(3) $\dfrac{2x^2+1}{9x^2-16}=\dfrac{x}{4+3x}-\dfrac{1}{9}$.

9. Solve the simultaneous equations

(1) $\left.\begin{aligned}\frac{4}{5+y}&=\frac{5}{12+x}\\ 2x+5y&=35\end{aligned}\right\}$ (2) $\left.\begin{aligned}3x+4y-5z&=15\\ 4x-5y+3z&=19\\ 5x-3y-4z&=46\end{aligned}\right\}$

10. A horse is sold at a loss for 42*l.*; had it been sold for 57*l.* the gain would have been four times the present loss: what did it cost?

11. The sum of two numbers exceeds three times their difference by 8, and their difference is equal to one-third of the lesser number: find them.

VII.

1. $x=5$, $a=8$. Find the value of
$a\sqrt{(x^2-3a)}+x\sqrt{(x^2+3a)}$.

2. Find the value of $\left(\frac{1}{2}+\frac{m-3}{2}\right)\left(\frac{1}{3}+\frac{m-2}{3}\right)$ and of
$(a^{\frac{4}{5}}+a^{\frac{3}{5}}x^{\frac{1}{5}}+a^{\frac{2}{5}}x^{\frac{2}{5}}+a^{\frac{1}{5}}x^{\frac{3}{5}}+x^{\frac{4}{5}})(a^{\frac{1}{5}}-x^{\frac{1}{5}})$.

3. Square $2m-n$, and extract the square root of
$9a^4-12a^3b+34a^2b^2-20ab^3+25b^4$.

4. Add together $1-[2-(3-x)]$, $3x-(4-5x)$, and $4-(-5+6x)$, and from $a-x-(x-2a)+2a-x$ take $a-2x-(2a-x)+(x-2a)$.

5. Solve

(1) $\frac{a}{b^2x}+\frac{b}{a^2x}=a^3+b^3$.

(2) $\frac{x}{4}-4\frac{1}{2}+\frac{x}{5\frac{1}{2}}+\frac{x}{2}=\frac{16\frac{1}{4}}{5\frac{1}{2}}$.

(3) $\left.\begin{aligned}\frac{x}{18}+\frac{y}{9}&=32\\ \frac{x}{9}+\frac{y}{18}&=28\end{aligned}\right\}$ (4) $\left.\begin{aligned}\frac{x}{a}+y&=m\\ \frac{y}{b}+x&=n\end{aligned}\right\}$

6. Divide the product of
$a+b-c$, $a-b+c$, and $b+c-a$ by $a^2-b^2-c^2-2bc$.

7. Find the L.C.M. of
$8x^3-14x+6$, $4x^2+4x-3$, and $4x^2+2x-6$.

8. Simplify

(1) $\dfrac{1}{x+\dfrac{1}{1+\dfrac{x+1}{3-x}}}$. (2) $1-\dfrac{1}{1+\dfrac{b}{a-b}}$.

9. A is twice as old as B; twenty-two years ago he was four times as old as B: what is A's age?

10. A and B together have 570*l.*; if A's money were three times what it is, and B's five times what it is, the sum would be 2350*l.*: what is the money of each?

VIII.

1. If $a=0$, $b=1$, $c=2$, $d=3$, find the value of

$$\frac{acd+2bc-d-\dfrac{2b}{3}+ad}{2c-3b}-\frac{2c+bd+2ac+b}{4a+cd+3bc}.$$

2. Multiply $x^2+\dfrac{3xy}{4}-y^2$ by $x^2-\dfrac{3xy}{4}+y^2$, and divide $4x^5-x^3+4x$ by $2x^2+3x+2$.

3. Find the square root of $\dfrac{4a^2}{b^2}+8+\dfrac{4b^2}{a^2}$, and of $(a+b)^2-c^2+(a+c)^2-b^2+(b+c)^2-a^2$.

4. What is the G.C.M. of $3a^3-7a^2b-33ab-7ab^2+11b^2+3b^3$ and $3a^2-10ab+3b^2$, and the L.C.M. of $(x+y)^2$, x^2-y^2, and $x-y$?

5. Resolve $a^2b^2-b^2c^2-c^2a^2+2abc^2$ into two factors.

6. Divide $\frac{a^3-3a^2b+3ab^2-b^3}{a^2-b^2}$ by $\frac{2ab-2b^2}{3}$, and multiply the answer by $\frac{a^2+ab}{a-b}$.

7. Simplify

$$\frac{3abc}{bc+ca-ab}-\frac{\frac{a-1}{a}+\frac{b-1}{b}+\frac{c-1}{c}}{\frac{1}{a}+\frac{1}{b}-\frac{1}{c}}.$$

8. Solve

(1) $x+\frac{3x-5}{2}=12-\frac{2x-4}{3}$.

(2) $(a+x)(b+x)=(c+x)(d+x)$.

(3) $\left.\begin{array}{l}\frac{1}{3}x+\frac{1}{2}y=\frac{4}{3}\\ \frac{1}{2}x+\frac{1}{3}y=\frac{7}{6}\end{array}\right\}$ (4) $\frac{1}{x}=\frac{2}{y}=\frac{20}{3(x+y)}$.

9. Find the number the third part of which added to its seventh part makes 20.

10. If A's money be increased by 36*s.* he will have three times as much as B; but if B's be diminished by 5*s.* he will have half as much as A: find the sum possessed by each.

IX.

1. Simplify

(1) $(2a-b)^2+2b(a+b)-3a^2-(a-b)^2+(a+b)(a-b)$.

(2) $\frac{1}{(x+1)(x+2)}-\frac{1}{(x+1)(x+2)(x+3)}$.

2. Multiply

(1) $m+n+p+q$ by $m-n-p-q$.

(2) $a-\frac{x^2}{a}$ by $\frac{a}{x}+\frac{x}{a}$.

3. Find the G.C.M. of a^4-x^4 and $a^4+a^3x-ax^3-x^4$, and the L.C.M. of $x-a$, $x+a$, x^2-a^2, x^2+a^2.

4. Extract the square root of

(1) $4x^4-12ax^3+25a^2x^2-24a^3x+16a^4$.

(2) a^2+x to four terms.

5. Square $x^{\frac{1}{2}}-x+\frac{1}{4}$, and $x^{\frac{1}{2}}-1+x^{-\frac{1}{2}}$.

6. Cube $3a-\frac{2}{3}b$, and $x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{\frac{1}{3}}$.

7. Reduce to their lowest terms

(1) $\frac{x^2+2x-3}{x^2+5x+6}$. (2) $\frac{3x^3-2x^2-x}{6x^2-x-1}$.

8. Find the value of

$$\frac{a(x^2+c^2)-2acx}{b(x^2+c^2)-2bcx}.$$

9. Solve the following equations:—

(1) $x+\frac{11-x}{3}=\frac{19-x}{2}$. (2) $\frac{a}{bx}+\frac{b}{ax}=a^2-b^2$.

(3) $\frac{\sqrt{x}+28}{\sqrt{x}+4}=\frac{\sqrt{x}+38}{\sqrt{x}+6}$. (4) $\left.\begin{aligned} 3x-\frac{y+7}{5}&=4 \\ 2y+\frac{x+7}{3}&=9 \end{aligned}\right\}$

10. A labourer was engaged for 36 days on condition that for every day he worked he was to receive 2*s.* 6*d.*, and for every day he was idle to forfeit 1*s.* 6*d.*; at the end of the time he received 2*l.* 18*s.*: how many days did he work?

11. Divide 64*l.* among 3 people, so that the first may have 3 times as much as the second, and the third $\frac{1}{3}$ as much as the first and second together.

X.

1. Simplify

$$\frac{1}{1+3a+2a^2}+\frac{1}{1-a-2a^2},$$

and

$$\frac{1}{1-x}-\frac{x}{1+x}.$$

2. Multiply $1-x-x^2$ by x^2-x-1,

and $x^{\frac{2}{3}}+x^{\frac{1}{3}}$ by $x-x^{\frac{2}{3}}$.

3. Divide $\dfrac{a^2x+ax^2+x^3}{a^3-x^3}$ by $\dfrac{x}{a-x}$,

and $a^{\frac{5}{6}}+a^{\frac{1}{2}}b^{\frac{1}{3}}+a^{\frac{1}{3}}b^{\frac{1}{2}}+b^{\frac{5}{6}}$ by $a^{\frac{1}{2}}+b^{\frac{1}{2}}$.

4. Find the G.C.M. of

$$x^3+5x^2+10x+8 \text{ and } x^5+2x^4-x-2,$$

and the L.C.M. of

$$1-x^2,\ 1-2x+x^2,\ 1-x+x^2-x^3,\ 1-x-x^2+x.$$

5. Extract the square root of

$$8x^2(2x^2+6ax+a)+a^2(36x^2+12x+1).$$

6. Reduce to their lowest terms

$$\frac{6a(a+b)-b(a+6b)}{4a(a+2b)+b(4a+9b)}, \qquad \frac{2x^2-xy-y^2}{y^2+xy-2x^2}.$$

7. Expand $(x^{\frac{1}{3}}-y^{\frac{1}{3}})^2$, $-\left\{\dfrac{ab^2c^3}{3}\right\}^5$, $\left(\dfrac{ab}{4}-\dfrac{1}{ab}\right)^3$.

8. Solve the equations

(1) $\frac{8x+3}{5}=12-\frac{4x-1}{3}$. (2) $4x=\frac{3cx+b}{c}$.

(3) $\frac{4}{x-3}+\frac{8}{x+1}=\frac{12}{x-1}$.

(4) $\left.\begin{aligned} 3x-\frac{5(y+2)}{2}&=4 \\ y+\frac{12x-16}{10}&=14 \end{aligned}\right\}$ (5) $\left.\begin{aligned} \frac{1}{y}+\frac{1}{z}-\frac{1}{x}&=6 \\ \frac{1}{z}+\frac{1}{x}-\frac{1}{y}&=4 \\ \frac{1}{x}+\frac{1}{y}-\frac{1}{z}&=2 \end{aligned}\right\}$

9. I have 1200 volumes in my library; twice as many duodecimos as octavos, and 5 times as many octavos as larger volumes: how many of each size do I possess?

10. We went up and came down a mountain 10000 feet high at an average pace of 1600 vertical feet per hour; but in descending we accomplished 7 feet in the time required for ascending 3: how long were we in reaching the top?

XI.

1. Simplify

(1) $\left(1+\frac{x}{y}\quad 1-\frac{x}{y}\right)\div x^2\frac{y}{+y^2}$.

(2) $1\frac{1}{2}-\frac{3}{4}\{1-\frac{2}{3}(x-\frac{1}{2})\}$.

2. Divide

(1) $a^3-\frac{1}{a^3}$ by $a-\frac{1}{a}$.

(2) $x^{\frac{5}{2}}-x^2-4x^{\frac{3}{2}}+6x-2x^{\frac{1}{2}}$ by $x^{\frac{3}{2}}-4x^{\frac{1}{2}}+2$.

3. Add together

$\frac{a^2}{2}-\frac{b^2}{3}+\frac{c^2}{4}, \frac{b^2}{2}-\frac{c^2}{3}+\frac{a^2}{4}, \frac{c^2}{2}-\frac{a^2}{3}+\frac{b^2}{4}$.

4. Find the G.C.M. of

$$20x^4+x^3-1 \text{ and } 25x^4+5x^3-x-1,$$

and the L.C.M. of

$$x^3+x^2y+xy^2+y^3 \text{ and } x^3-x^2y+xy^2-y^3.$$

5. Extract the square root of

$a^2+4b^2+c^2+2ac+4ab+4bc$, and of $x^2+\frac{y}{2}$ to 3 terms.

6. Reduce to their lowest terms

(1) $\frac{2x^2-3x-2}{5x^2-7x-6}$, (2) $\frac{x^3-3x^2+7x-21}{2x^4+19x^2+35}$.

7. Expand $(a^{\frac{1}{2}}+b^{\frac{1}{2}}+c^{\frac{1}{2}})^2$ and $\left(\frac{x}{6}+\frac{6}{y}\right)^2$.

8. Solve the equations

(1) $\frac{5x}{9}-8=74-\frac{7x}{12}$.

(2) $\frac{42}{x-2}=\frac{35}{x-3}$. (3) $\frac{a}{x-b}=\frac{b}{x-a}$.

(4) $\frac{ax-1}{\sqrt{ax}+1}=4+\frac{\sqrt{ax}-1}{2}$.

(5) $\left.\begin{matrix}5x-3y-28=0\\3x-2y-13=0\end{matrix}\right\}$

9. Divide the number 90 into 4 parts, so that the first part increased by 2, the second diminished by 2, the third multiplied by 2, and the fourth divided by 2 may be all equal.

10. A greyhound starts for a course at a distance of fifty of his own strides from the hare; he makes three *strides for* every four of the hare's, and passes over as

much ground in two strides as the hare does in three: how many strides does each make before the hare is caught?

XII.

1. Add together

$1-(1\quad 2-6x)$, $2x-(3-5x)$, and $2+(-4+5x)$.

2. Multiply

a^6+a^5-a-1 by $1-a+a^2-a^3+a^4$,

and $x^{(m-1)n}-y^{(n-1)m}$ by x^n-y^m.

3. Divide

$x^{\frac{5}{2}}-x^2-4x+6x-2x^{\frac{1}{2}}$ by $x-x^{\frac{1}{2}}$.

4. Find the G.C.M. of

$4x^4+22x^3-26x^2-198x-90$

and $4x^3-14x^2-92x-42$.

5. Find the L.C.M. of

x^2-1, x^3-1, x^3+1, $(x+1)^2$, $(x-1)^2$.

6. Find the square root of $x^4+2a^2+\frac{a^4}{x^4}$,

and the cube root of $x^6+9x^4+27x^2+27$.

7. What is the value of $\left\{\left(\frac{1}{64}\right)^{\frac{2}{3}}\right\}^{-\frac{1}{2}}$?

8. Solve the equations

(1) $(x-3)(7x-4)=(7x-26)(x-1)$.

(2) $\frac{x}{a}+\frac{x}{b}+\frac{x}{c}=k$.

(3) $\left.\begin{aligned} \frac{x}{7}+7x&=99 \\ \frac{y}{7}+7x&=51 \end{aligned}\right\}$ (4) $\left.\begin{aligned} x+2z+3y&=11 \\ 2x+3z+y&=14 \\ 3x+2y+z&=11 \end{aligned}\right\}$

9. What sum is that which is as much greater than $2l.$ as its half is less than $3l.$?

10. The united ages of a father and son make 75 years; the father was 25 years old when the son was born: what is the age of each?

11. A number consisting of two digits, when divided by the sum of the digits, gives a quotient 7; and the number formed by inverting the digits, when divided by 6, gives a quotient 4: what is the number?

12. If $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$, prove that

$$\frac{x+y+z}{a+b+c}=\frac{(b+c)x+(c+a)y+(a+b)z}{2(a^2+b^2+c^2)}.$$

XIII.

1. Explain the meaning of the following algebraical terms—binomial, factor, surd, power, index.

2. Find the value of

$$x-(\sqrt{x+1}+2)-\frac{x-\sqrt[3]{x}}{\sqrt{x}-4} \text{ when } x=8.$$

3. Multiply

$$x^{\frac{3}{2}}y^{-\frac{1}{2}}-x^{-\frac{1}{3}}y^{\frac{1}{2}} \text{ by } x^{\frac{1}{3}}y^{\frac{1}{2}}+x^{-\frac{1}{3}}y^{\frac{3}{2}};$$

and divide

$$a^2x^4-c^2y^4-bx^2y(2ax-by) \text{ by } (ax-by)x-cy^2.$$

4. Find the G.C.M. of

$3ax^2+2ay^2-3bx^2-2by^2$ and

$3ax^2-2ay^2-3bx^2+2by^2$,

and the L.C.M. of

a^2+x^2, a^2-x^2, a^3+x^3, and a^3-x^3.

5. Reduce to simple fractions:

(1) $\dfrac{\dfrac{a+x}{a-x}-1}{\dfrac{a-x}{a+x}+1}$.

(2) $\dfrac{x}{a-\dfrac{x}{a-\dfrac{x}{a}}}$

(3) $\dfrac{ay}{x}+\dfrac{bx}{y}-\dfrac{ay+bx}{2xy}$.

(4) $\dfrac{a^4-x^4}{a+3}\times\dfrac{1}{a^2-2x-a(a-2)}\times\dfrac{a^2+5a+6}{(a^2+x^2)(a+x)}$.

6. Find the square root of

$$\frac{4x^4-12x^3y+x^2y^2+12xy^3+4y^4}{16x^2y^2}.$$

7. Shew that the product of any three consecutive numbers is equal to the difference between the middle number and its cube.

8. A is 5 years more than twice as old as B, and is also 25 years older than B: find their present ages.

9. When are the hour and minute hands of a clock in a straight line, in conjunction or opposite to one another, between the hours of 2 and 3?

10. Solve the following equations:—

(1) $3x-a=\dfrac{a+x}{3}-\dfrac{b-x}{a}-cx$.

(2) $\frac{x^3}{2}+\frac{x^2}{3}+\frac{x}{4}=\frac{x(x+2)(x-2)}{2}+\frac{x^3+7}{3}$.

(3) $\left.\begin{array}{l}\frac{x-y}{6}=2\\ \frac{3x-7y}{3}=\frac{2x+y+1}{5}\end{array}\right\}$ (4) $\left.\begin{array}{r}bx+ay=ab\\ cy+bz=bc\\ az+cx=ac\end{array}\right\}$

11. A number consists of two digits, the sum of which is 7; if 27 be subtracted from the number, the order of the digits will be inverted: find the number.

XIV.

1. What is the value of a^0, a^1, $\frac{0}{a}$, a^{-2},

and of $\sqrt[3]{x}\left\{y^3+\sqrt[4]{\frac{x^2+2xy+y^2}{y^2}}\right\}$

when $x=8$, $y=1$?

2. Multiply $x^3+3ax(x+a)+a^3$ by $x-a$,
and divide $(a^2-2ab+b^2)(x^3+a^3)$ by $ax-bx+a^2-ab$.

3. Find the G.C.M. of
$2x^2+3xy+6x+9y$ and $3x^2-2xy+9x-6y$.

4. Find the L.C.M. of
x^2-1, x^2-4y^2, and $x^2+x-2xy-2y$.

5. Simplify

(1) $\dfrac{x}{1+\dfrac{x}{1+x+\dfrac{x}{1+x+x^2}}}$.

(2) $\frac{x^4-y^4}{x^2-2xy+y^2}\div\frac{x^2y+y^3}{x^3-y^3}$.

6. Find the square root of

$$\left(x^2+\frac{x^2}{y}-2x\right)\left(1+\frac{1}{y}\right)+1.$$

7. Find two numbers in the proportion of 2 to 3, the sum of whose squares is 117.

8. Solve the following equations:—

(1) $\frac{1}{12}x-\frac{1}{8}(8-x)-\frac{1}{4}(5+x)+\frac{11}{4}=0.$

(2) $\frac{(2x+3)x}{2x+1}+\frac{1}{3x}=x+1.$

(3) $\left.\begin{matrix}x+ay=b\\ ax-by=c\end{matrix}\right\}$ (4) $\left.\begin{matrix}2x+5y=2\frac{1}{2}\\ 5x+2y=2\frac{1}{20}\end{matrix}\right\}$

9. A had twice as much money as B; but when B had won 5*l.* from A, he had three times as much as A: how much had they each at first?

10. Find a fraction such that if 5 be added to the numerator it $=1$, but if 3 be taken from the denominator it $=\frac{1}{2}$.

XV.

1. Find the value of

$3x^2-2\{x+(1-x)+2x\{1-(3-2x)\}$ when $x=1$,

and of

$$\frac{a^2+ac+b^2}{a^2-ac+b^2}-\frac{\sqrt{4ab+b^2+d}}{\sqrt{4ab-b^2-2d}}-\frac{c}{a+b+c+d}$$

when $a=4$, $b=3$, $c=1$, $d=7$.

2. Find the G.C.M. of

x^4+3x^3-6x-4 and x^4-3x^3+6x-4.

3. What is meant by L.C.M., and what is its principal use? Find the L.C.M. of

(1) x^3-a^3, x^3+a^3, $(x-a)^2$, $(x+a)^2$.

(2) $9a^2-81$, $12a^2-36a$, and $16a^2+48a$.

4. Simplify the following fractions:—

(1) $\frac{1}{x-y}+\frac{1}{x+y}-\frac{2}{x}$.

(2) $\frac{x^3-3ax^2+3a^2x-a^3}{x^2-a^2}\div\frac{x-a}{x+a}\times\frac{(x+a)^2}{2}\div\frac{x^2+2ax+a^2}{x-a}$.

5. Find the square root of $a^2-4ax+6a+4x^2-12x+9$.

6. Solve the following equations:—

(1) $\frac{x}{a+b}+\frac{x}{a-b}=\frac{4}{a^2-b^2}$.

(2) $\frac{12x-2}{6}-(x+2)=\frac{18-4x}{3}$.

7. When are the hour and minute hands of a watch together between 10 and 11 o'clock?

8. Divide 86 into two parts such that their difference is 18.

9. Solve the following equations:—

(1) $\left.\begin{matrix}5x+\frac{9}{2}y=52\\ \frac{3}{5}x+4y=27\end{matrix}\right\}$ (2) $\left.\begin{matrix}\frac{x}{a}+\frac{y}{b}=2\\ bx-ay=0\end{matrix}\right\}$

10. Find the fraction which if the numerator be increased by 2 is equal to 1, but if the denominator is increased by 1 it is equal to $\frac{1}{2}$.

XVI.

1. Subtract

$a^2-13ab+10ac-c^2$ from $7a^2+12ab-3ac-b^2$,

and find the value of the remainder when

$a=1$, $b=2$, $c=-3$.

2. Divide

(1) $a^2-4ab+4b^2$ by $\dfrac{a^2-2ab}{a+2b}$.

(2) $x^5-\dfrac{1}{x^5}$ by $x-\dfrac{1}{x}$. (3) $x^{\frac{1}{2}}y^{-\frac{1}{2}}$ by $x^{-\frac{1}{2}}y^{\frac{1}{2}}$.

3. Simplify

(1) $\dfrac{m+n}{m-n}\div\left[\dfrac{(m+n)^3-(m-n)^3}{(m^2-n^2)(3m^2+n^2)}\right]$.

(2) $\left\{\dfrac{x^2+a^2}{x^2y^2}+\sqrt{\dfrac{8a^3x^3}{x^6y^6}}\right\}^{\frac{1}{2}}\times\dfrac{x^2y-xy^2}{x^2+xa-xy-ay}$.

4. Find the G.C.M. of

$96x^4+8x^3-2x$ and $32x^3-24x^2-8x+3$,

and the L.C.M. of

$(a+x)^2$, $(a-x)^2$, a^2-x^2, and a^3-x^3.

5. Extract the square root of

$a^2-4ab^2-6ac^3+4b^2(b^2+3c^3)+9c^6$.

6. Add together

$\dfrac{1}{1+\dfrac{1}{1+x}}$, $\dfrac{x-(1-x)}{x+3-(1-3x)}$, and $\dfrac{10x^2-x-3}{4x^2+4x+1}$.

7. Solve the equations

(1) $\dfrac{x}{11}+1=\dfrac{1}{10}(x-1)$. (2) $\dfrac{a^2}{x}-\dfrac{b^2}{x}=a-b$.

(3) $ax^2+bx+c=a(1-x)^2+b(1-x)-c.$

(4) $\frac{3x-1}{11}+\frac{5x+3}{23}-\frac{x-3}{4}=2+\frac{x}{16}.$

8. The sum of two numbers is 1000, and their difference is $\frac{1}{12}$ of the less: find them.

9. Divide 20000*l.* between A, B, C, so that A's share is $\frac{1}{5}$ of C's, and A's and B's together equal to C's.

XVII.

1. Find the value of

$$\frac{3}{1+x}+\frac{3}{1-x}, \text{ when } x=\tfrac{1}{2},$$

and of x^3-2xy^2+y-13, when $x=2$ and $y=-3$.

2. Simplify

$$a+b-(2a-3b)-(5a+7b)-(-13a+2b)$$

and $\sqrt{ax^2-6ax+9a}$.

3. Multiply

(1) $x^2-\frac{3}{4}x+1$ by $x^2-\frac{1}{2}x$.

(2) x^3-x^{-3} by $x-x^{-1}$.

4. Divide

$a^3+b^3+c^3-3abc$ by $a+b+c$.

5. Simplify

(1) $\frac{a}{2a-2b}+\frac{b}{2b-2a}$.

(2) $\left\{\frac{x}{1+x}+\frac{1-x}{x}\right\}+\left\{\frac{x}{1+x}-\frac{1-x}{x}\right\}$.

(3) $\frac{a-\sqrt{b}}{c+\sqrt{d}}+\frac{a+\sqrt{b}}{c-\sqrt{d}}$.

6. Find the G.C.M. of

$$48x^2+16x-15 \text{ and } 24x^3-22x^2+17x-5,$$

and the L.C.M. of

$$2x-1,\ 4x^2-1,\ 4x^2+1.$$

7. Extract the square root of

(1) $9a^4+34a^2b^2-20ab^3-12a^3b+25b^4$,

(2) $x^4-2x^3+\frac{3x^2}{2}-\frac{x}{2}+\frac{1}{16}$;

and the cube root of

$$8x^3-12(a-1)x^2+6(a^2-2a+1)x-(a^3-3a^2+3a-1).$$

8. Solve the equations

(1) $2x-\frac{x+3}{3}+15=\frac{12x+26}{5}$.

(2) $\frac{5x+3}{x-1}+\frac{2x-3}{2x-2}=9$.

(3) $\frac{x}{a-x}=\frac{b-x}{x}$.

(4) $\left.\begin{aligned}\frac{x+y}{3}-2y&=2\\ \frac{2x-4y}{5}+y&=\frac{23}{5}\end{aligned}\right\}$

(5) $\left.\begin{aligned}x+2y+3z&=10\\ 2x+3y+4z&=16\\ 4x+4y+5z&=25\end{aligned}\right\}$

9. A post is $\frac{1}{8}$ in the earth, $\frac{3}{7}$ in the water, and 13 feet out of the water: what is the length of the post?

10. A vessel containing 120 gallons is filled in 10 minutes by two pipes running *successively*; the one pipe runs 14 gallons in a minute, and the other 9 gallons: for what time did each run?

11. In betting, a man on the first day lost $\frac{1}{2}$ of his money, and then won 10*s.*; on the next day he lost $\frac{1}{3}$ of the remainder, and then won 3*s.*; after which he had 3 guineas left: what money had he at first?

XVIII.

1. Add together

$$1-\{\tfrac{1}{2}-(\tfrac{1}{4}-x)\},\ \tfrac{1}{2}-(\tfrac{1}{3}-5x),\ \tfrac{1}{3}-(\tfrac{1}{4}+4x).$$

2. Give the value of

(1) $2a\sqrt{b^2-ac}+\sqrt{2ac+c^2}$

when $a=6$, $b=5$, $c=4$.

(2) $\dfrac{x}{y}-\sqrt{\dfrac{1-x^2}{1-y}}$ when $x=\frac{1}{4}$, $y=\frac{1}{8}$.

3. Multiply

(1) $x^2-\frac{3}{4}x+1$ by $x^2-\frac{1}{2}x$. (2) x^m+x by x^n.

(3) $(x+y)^2-(x+y)+1$ by $x+y+1$.

4. Divide

(1) $x^4-\frac{13}{6}x^3+x^2+\frac{4}{3}x-2$ by $\frac{4}{3}x-2$.

(2) $a+b$ by $a^{\frac{1}{3}}+b^{\frac{1}{3}}$.

5. Find the G.C.M. of

(1) $20x^4-x^2-1$ and $25x^4-5x^3-x-1$,

(2) x^4-1 and x^5+x^3;

and the L.C.M. of

$$a^3-a^2b-ab+b^2 \text{ and } a^4-b^2.$$

6. Simplify

(1) $\dfrac{m^2-n^2}{3m-3n}$. (2) $\dfrac{x^2+x-2}{2x^2-3x+1}$.

(3) $\dfrac{4x^4-1}{9x^6-y^2}\times\dfrac{3x^3-y}{2x^3-1}$. (4) $\dfrac{a+1}{b}\div\dfrac{1\times\frac{1}{a}}{b}$.

(5) $3\sqrt{8}\times 2\sqrt{6}$.

7. Extract the square root of

(1) a^2+b to 3 terms.

(2) $m^2+2m-1-\frac{2}{m}+\frac{1}{m^2}$.

8. Solve

(1) $\frac{x-3}{2}-\frac{x+\frac{3}{2}}{2}=\frac{5-\frac{1}{2}x}{4}$.

(2) $\cdot 75x+2\cdot 5=\cdot 5x+4\cdot 5$.

(3) $\left\{\begin{array}{l}\frac{3x-7y}{3}=\frac{2x+y+1}{5}\\ 8-\frac{x-y}{5}=6\end{array}\right\}$

9. Divide 560*l.* between A and B so that for every halfcrown A receives B shall receive a shilling.

10. Out of a cask a third part had leaked away; 21 gallons were then drawn, and the cask was still half full: how much did it hold at first?

11. A tenant takes a farm for 175*l.* 15*s.* per annum; he gives 2*l.* per acre for the arable part, and for the remainder, 5 acres less, and not arable, he pays 1*l.* 5*s.* per acre: how many acres are arable, and how many not?

12. Prove that

(1) $(a^2+b^2+c^2)(x^2+y^2+z^2)-(ax+by+cz)^2$
$=(cy-bz)^2+(az-cx)^2+(bx-ay)^2$;

(2) $(a+b+c)(-a+b+c)+(a-b+c)(a+b-c)$
$=4bc$;

(3) if $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$.

XIX.

1. Simplify $1-\{1-(1-\overline{1-x})\}$ and $\frac{1}{6}\{x(x+1)(x+2)+x(x-1)(x-2)\}+\frac{2}{3}(x-1)x(x+1)$.

2. What is the value of

$$5\sqrt{62+3x}-\tfrac{1}{2}\sqrt{95\tfrac{2}{3}-5x} \text{ when } x=6\tfrac{1}{3}?$$

3. Multiply

(1) $x^2-\frac{1}{2}x+\frac{2}{3}$ by $\frac{1}{3}x+2$.

(2) x^2-px+q by $x+a$, bracketing the coefficients of x.

(3) $x+2y^{\frac{1}{3}}+3z^{\frac{1}{3}}$ by $x-2y^{\frac{1}{3}}+3z^{\frac{1}{3}}$.

4. Divide

(1) $x^4+10x^3+35x^2+50x+24$ by the product $(x+1)(x+4)$.

(2) $a-b$ by $\sqrt[4]{a}-\sqrt[4]{b}$.

5. Find the G.C.M. of

$6a^3-6a^2y+2ay^2-2y^3$ and $12a^2-15ay+3y^2$.

6. Find the L.C.M. of

(1) $x^3-4x^2+9x-10$ and $x^3+2x^2-3x+20$.

(2) x^2-1, $(x+1)^2$, x^3-1.

7. Simplify

(1) $\dfrac{4m-3n}{3(1-n)}-\dfrac{m+3n}{3(1-n)}+\dfrac{2n}{1-n}$

(2) $\dfrac{a+\dfrac{b-a}{1+ba}}{1-a\dfrac{b-a}{1+ba}}$.

(3) $\dfrac{1}{x-1+\dfrac{1}{1+\dfrac{x}{4-x}}}$.

(4) $\dfrac{x^3+xy}{x-y}+\dfrac{x^4-y^4}{(x-y)^2}$.

(5) $\dfrac{\sqrt{1-x}+\dfrac{1}{\sqrt{1+x}}}{1+\dfrac{1}{\sqrt{1-x^2}}}$. (6) $\sqrt[3]{4}\times 7\sqrt[3]{6}\times \frac{1}{2}\sqrt[3]{5}$.

8. Extract the square root of

(1) $x^2-2ax+a^2+2x-2a+1$.

(2) $\dfrac{a^2}{b^2}+\dfrac{b^2}{a^2}+\dfrac{2a}{b}+\dfrac{2b}{a}+3$.

9. Solve

(1) $\dfrac{x-3}{2}-\dfrac{x+\frac{3}{2}}{2}=\dfrac{5-\frac{7}{2}x}{4}$.

(2) $12-x : \dfrac{x}{2} :: 4 : 1$. (3) $\dfrac{x+a}{x}+\dfrac{2x}{x+a}=3$

(4) $\left.\begin{aligned}\frac{x}{3}+\frac{y}{5}-5&=0\\ 2x+\frac{y}{3}-17&=0\end{aligned}\right\}$ (5) $\left.\begin{aligned}z+y&=a\\ z+x&=b\\ y+x&=c\end{aligned}\right\}$

(6) $\sqrt{4x+9}-2\sqrt{x}=1$.

10. At an election 1296 persons voted; the successful candidate had a majority of 120: how many voted for each?

11. A travels $6\frac{1}{2}$ miles an hour, and starts $2\frac{1}{2}$ hours before B, who travels the same road at the rate of 9 miles an hour: how long will A travel before he is overtaken by B?

12. A and B have both the same income; A saves $\frac{1}{5}$ of his yearly, while B, by spending 50*l.* a year more than A, at the end of 4 years finds himself 100*l.* in debt: what is their income

XX.

1. If $a=3$, $b=1$, $c=-2$, $d=0$, find the numerical values of the two following expressions :—

(1) $$\frac{a^3-b^3}{2ab+2bc+2cd+2da} + \frac{a^2+b^2+c^2+d^2}{\sqrt{\{(a-c)(b+c)+6(-c-b)(a-2b)\}}}.$$

(2) $$\sqrt[3]{c^3-3c^2a+3a^2c-27} + \sqrt{4a^2d^4-(a-d)(c-b)(b+d)}.$$

2. Remove the brackets from

(1) $x^5+7x^4-6x^3+5x^2-4x-3-(x^5-2x^4+3x^2+4x-5)$

(2) $10x-\{3x-[-4x-(-2x+3x)]\}$.

3. Divide $(a^2+2bc)^3-4bc(3a^4+4b^2c^2)$ by a^2-2bc.

4. Find the G.C.M. of

$6x^3-x^2+16$, and $6x^4-3x^3+9x^2+3x+12$,

and the L.C.M. of x^3+x, $1-x$, x^2-1.

5. Simplify the following expressions :—

(1) $$\frac{x^2+x+1}{x-1+\frac{1}{x}} \times \frac{\frac{1}{x^2}-\frac{1}{x}+1}{x+1+\frac{1}{x}}.$$

(2) $$\frac{m}{n+\frac{p}{q+\frac{r}{s-t}}}.$$

(3) $$\frac{a-\frac{b^2}{a+b}}{b+\frac{a^2}{a-b}}.$$

(4) $$\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+x+1}.$$

6. Square $ax^2+b^2x-c^3$, and find the square root of $x^6-4x^5+10x^4-16x^3+17x^2-12x+4$.

7. Solve the following equations :—

(1) $\dfrac{10x-7}{2x-1}-\dfrac{9x-1}{3x+1}=2.$

(2) $\frac{1}{8}(12-x)+\dfrac{3x-\frac{1}{2}}{3}=\dfrac{4x-\frac{2}{3}}{2}+\dfrac{3\frac{1}{6}-x}{7}.$

(3) $x+\dfrac{a}{b-a}=\dfrac{bx}{a+b}.$

(4) $\left.\begin{aligned}2x+3y+4z&=34\\3x-2y+2z&=13\\y-z&=1\end{aligned}\right\}$

(5) $\left.\begin{aligned}\frac{1}{y}+\frac{1}{z}&=7\\\frac{1}{z}+\frac{1}{x}&=6\\\frac{1}{x}+\frac{1}{y}&=5\end{aligned}\right\}$

8. Ten years ago a father was three times the age of his son ; in two years' time he will be twice the age : what are their respective ages now ?

9. Find two consecutive numbers the difference of the squares of which is equal to 51.

10. Two men run a mile against one another, the winner finishing in 4 minutes 44 seconds, and winning by 2 seconds : how much start does the loser require for a tie, supposing each of the two to run on all occasions at one uniform pace ?

11. A boat's crew rows to Iffley and back, the current flowing at the rate of 3 miles an hour : their pace on still water being 9 miles an hour, how does the current affect the time of the double journey ?

12. Find the value of $\dfrac{x^2-xy+y^2}{x^2+xy+y^2}$

when $x=\dfrac{\sqrt{3}+1}{2}$, $y=\dfrac{\sqrt{3}-1}{2}$.

XXI.

1. Find the value of

$$\sqrt[5]{32}+\sqrt[3]{1}-2\sqrt{4}+\sqrt[4]{81},$$

and of $\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{f}$

when $a=0$, $b=1$, $c=-2$, $d=4$, $f=\frac{1}{2}$.

2. Prove that

(1) $(x^2+y^2)(a^2+b^2)-(ax+by)^2=(bx-ay)^2$.

(2) $(a-b)^2+\dfrac{6a^2b+2b^3}{a-b}=\dfrac{(a+b)^3}{a-b}$.

3. Find the G.C.M. of

$$4x^3+4x^2-9x-9 \text{ and } x^3+2x^2-x-2,$$

and the L.C.M. of

$$x^2-x,\ x^2-1,\ x^3-2x^2, \text{ and } x^3-4x.$$

4. Simplify the following expressions:—

(1) $\dfrac{\dfrac{1}{a+b}+\dfrac{1}{a-b}}{\dfrac{1}{a-b}-\dfrac{1}{a+b}}\div\dfrac{a}{b}$.

(2) $\dfrac{1}{1-\dfrac{1}{1+\dfrac{1}{x}}}$.

(3) $\left(\dfrac{x^2-y^2}{x^2-2xy+y^2}\div\dfrac{x^2+3x}{x-3}\right)$
$\times\left(\dfrac{x^5-9x^3}{x+y}\div\dfrac{x^3-3x^2}{x-y}\right)$.

5. Solve the following equations:—

(1) $\frac{1}{2}(x-1)-\frac{1}{3}(2-x)+\frac{1}{4}(x+1)=x$.

(2) $\left.\begin{matrix}\frac{1}{2}x+\frac{1}{3}y=16\\ \frac{1}{3}x-\frac{1}{2}y=2\end{matrix}\right\}$ (3) $\left.\begin{matrix}x-a=y+b\\ x+y=0\end{matrix}\right\}$

6. Find the square root of

(1) $a^2+4b^2+9c^2-4ab+6ac-12bc$.

(2) $x^4-2x^3+\frac{3x^2}{2}-\frac{x}{2}+\frac{1}{16}$.

7. Divide 13*l.* among three people so that the first may have twice as much as the second, and the third 3*l.* less than the first.

8. Find the number consisting of two digits whose sum is 9 such that if the order of the digits be reversed the number so obtained : the original number :: 7 : 4.

9. Simplify

$$\{x^2+(\sqrt{3}-1)x-\sqrt{3}\}\{x^2-(\sqrt{3}-1)x-\sqrt{3}\}.$$

XXII.

1. Find the value of $\frac{a^2+b^2}{a^3-b^3}$ when $a=3$, $b=-2$;

of $\frac{a+\sqrt{a^2+b^2}}{a^3-2b(a^2-b^2)}$ when $a=-4$ and $b=-3$.

2. Bracket together the equivalent quantities in this list:—

$$\frac{1}{a^3},\ a^2,\ a^{\frac{1}{2}},\ a^{-3},\ \sqrt{a},\ \sqrt{a^4},\ a^{\frac{4}{3}},\ \sqrt[6]{a^3},\ \frac{a^4}{a^7},\ a^3\times a^{-1};$$

and simplify

$$1-\{1-(2-x)\}+\{4x-(3-6x)\}-\{4+(6x-5)\}.$$

3. Find the G.C.M. of

$3x^3+5x^2-x+2$ and $4x^4+9x^3+2x^2-2x-4$;

and the L.C.M. of

$$(a-1),\ (a^2-1),\ (a-2),\ (a^2-4).$$

4. Simplify $\dfrac{a+b+\dfrac{b^2}{a}}{a+b+\dfrac{a^2}{b}}$ and $\dfrac{1}{a+\dfrac{1}{b+\dfrac{1}{c}}}$.

5. Multiply $x+1+\frac{1}{x}$ by $x-1+\frac{1}{x}$;

and divide $x^4-\frac{1}{x^4}$ by $x-\frac{1}{x}$.

6. Simplify $\dfrac{x^2+5x+4}{x^2+7x+12} \div \dfrac{x^2+2x+1}{x^2+3x+2}$,

and $\dfrac{2x^2}{a^3+x^3} \div \dfrac{x}{a+x}$.

7. The square root of

$$9a^4-12a^3b+34a^2b^2-20ab^3+25b^4.$$

8. Solve

(1) $5x-\dfrac{2x-1}{3}+1=3x+\dfrac{x+2}{2}+7.$

(2) $\frac{1}{12}x-\frac{1}{8}(8-x)-\frac{1}{4}(5+x)+\frac{11}{4}=0.$

(3) $(x+5)(y+7)=(x+1)(y-9)+112;$
$2x+10=3y+1.$

(4) $a+x+\sqrt{2ax+x^2}=b.$

9. Find two numbers, the sum of which is 15, and that of their squares 113.

10. A person at billiards bet 3*l.* to 2*l.* on every game; after playing 20 games he won 5*l.*: how many games did he win?

11. Two goods trains pass a station at an interval of 4 hours, travelling respectively at the rate of $11\frac{1}{2}$ and $17\frac{1}{2}$ miles per hour: how far will the slower train have *run before it is* overtaken by the other?

XXIII.

1. If $a+b=5$ and $a-b=1$, find the value of

$$\frac{2a+3b+2ab}{\frac{(a+3b)}{a}+\frac{(a+b)(a-b)}{5}}.$$

2. Multiply $a^4+a^3x+x^{\frac{3}{2}}$ by $a^{\frac{3}{2}}+ax^3+x^4$.

3. Divide $x^4y^3-3x^3y^3+3x^2y^3-x^3y^4$ by $-x^2y^2$; and a^4-16 by $a-2$.

4. Find the square root of

$$16m^4+\tfrac{16}{3}m^2n+8m^2+\tfrac{4}{9}n^2+\tfrac{4}{3}n+1.$$

5. Find the G.C.M. of

$$x^4-ax^3+a^3x-a^4 \text{ and } x^3-a^3,$$

and the L.C.M. of

$$(b^2y-by^2)^2 \text{ and } by(b^2-y^2)^2.$$

6. Simplify

$$\frac{am^2+an^2+m^2n^2+m^4}{bm^2+bn^2+m^2n^2+n^4};$$

what would be the result if a were $=-m^2$?

7. Find the value of $\sqrt[3]{64x^3y^6z^9}+\sqrt[4]{x^4y^8z^{12}}$; and of $(2mnp-q)^2$.

8. Solve the following equations:—

(1) $\frac{x^2}{2}-3x=\frac{x(a+x)}{2}-\left(\frac{ax}{2}+12\right)$;

(2) $\frac{4(x+1)+4(x-1)}{(x+1)(x-\ )}=\frac{x+7}{x^2-1}$;

(3) $\dfrac{a+b-x}{2}+\dfrac{c+d-x}{3}+\dfrac{e+f-x}{4}+x=0$;

(4) $\left.\begin{aligned}\frac{x}{2}+\frac{y}{3}&=7\\ \frac{x}{3}-\frac{y}{8}&=\tfrac{1}{2}\end{aligned}\right\}$ (5) $\left.\begin{aligned}\frac{2}{x}+\frac{3}{y}&=5a-b\\ \frac{3}{y}-\frac{2}{x}&=a-5b\end{aligned}\right\}$.

9. A grocer would mix spice at 10*s.* a pound with another sort at 6*s.* a pound, to make 100 lb. to be sold at 7*s.* a pound. What quantity of each must he take?

10. The difference between a number and its square is 18 more than the same difference would be if the number were 1 less. Find the number.

XXIV.

1. Find the value of

$$ax+by \text{ when } x=\frac{cq-br}{aq-bp} \text{ and } y=\frac{ar-cp}{aq-bp},$$

and of x^3-2xy^2+y-13 when $x=2$ and $y=-3$.

2. Simplify $3x^2+3y^2-\{x^2+2xy+y^2-(2xy-x^2-y^2)\}$.

3. Add together

$$x^2-3xy-\tfrac{2}{3}y^2,\ 2y^2-\tfrac{2}{3}y^3+z^2,\ xy-\tfrac{1}{3}y^2+y^3,\ 2xy-\tfrac{1}{3}y^3.$$

4. Reduce to its lowest terms

$$\frac{1}{4a^3(a+x)}+\frac{1}{4a^3(a-x)}+\frac{1}{2a^2(a^2+x^2)}.$$

5. Multiply $x^2-\frac{1}{2}x+\frac{2}{3}$ by $\frac{1}{3}x+2$,

and divide $x^3-c\frac{5}{2}x^2+a^2px-a^3$ by $x-a$.

6. Find the G.C.M. of

$$20x^4+x^2-1 \text{ and } 25x^4+5x^3-x-1,$$

and the L.C.M. of

$$x^2+5x+4,\ x^2+2x-8,\ x^2+7x+12.$$

7. Extract the square root of

$$a^4-a^2b+2a^2b^2+\tfrac{1}{4}b^2-b^3+b^4.$$

8. Solve the following :—

(1) $(m+n)(m-x)=m(n-x)$.

(2) $\dfrac{6x+13}{15}-\dfrac{3x+5}{5x-25}=\dfrac{2x}{5}$.

(3) $\dfrac{x+1\frac{1}{2}}{3}-\dfrac{10-x}{3\frac{2}{3}}=\dfrac{4-\frac{2}{3}x}{11}-\dfrac{1}{11}$.

(4) $\sqrt{4x+9}-2\sqrt{x}=1$.

(5) $$\left.\begin{aligned} x+1-\frac{3y+4x}{7}&=7-\frac{9y+33}{14}\\ y-3-\frac{5x-4y}{2}&=x-\frac{11y-19}{4}\end{aligned}\right\}$$

9. Find two numbers whose difference is 1, and the sum of their squares amounts to 313.

10. A man agreed to work for 30 days, on condition that for every day's work he should receive 3*s.* 4*d.*, and that for every day's absence from work he should forfeit 1*s.* 6*d.*; at the end of the time he received 3*l.* 11*s.*: how many days did he work, and how many was he absent?

11. If $s=\dfrac{a+b+c}{2}$, shew that

(1) $s^2+(s-a)^2+(s-b)^2+(s-c)^2=a^2+b^2+c^2$;

(2) $(s-a)^3+(s-b)^3+(s-c)^3+3(s-a)\ (s-b)c$
$=\{(s-a)+(s-b)\}^3$.

XXV.

1. Remove the brackets from

$$x-[3ax-\{2x-(a-2x)b-3a(b-x)\}+2b(x-2a)].$$

2. Find the value of

$$\frac{a(bc-de)-b(ad-ce)}{c(ab-de)-d(ac-be)},$$

when $a=1$, $b=-1$, $c=2$, $d=-2$, $e=0$.

3. Add together

$$a(a-b+c),\ b(b-c+a),\ c(c-a+b);$$

and subtract $\frac{1-3x}{1+3x}$ from $\frac{3x}{1-3x}$.

4. Find the G.C.M. of

$$a^3-2ab^2-b^3 \text{ and } a^3+2a^2b+2ab^2+b^3;$$

and the L.C.M. of

$$a-b,\ a+b,\ a^2-b^2,\ a^3-b^3,\ a^3+b^3.$$

5. Write down the square of $\frac{x}{4}-\frac{4}{x}$; and find the square root of

$$9a^2-12ab+24a+4b^2-16b+16.$$

6. Simplify the expressions

$$(1)\ \frac{\frac{x^2+y^2}{y}-x}{\frac{1}{y}-\frac{1}{x}}\times\frac{x^2-y^2}{x^3+y^3}. \qquad (2)\ 1+\frac{x}{1+x+\frac{2x^2}{1-x}}.$$

7. Prove that the difference of the squares of any unequal numbers is equal to the product of their sum and difference.

Resolve $x^3-2\frac{1}{16}$ into rational factors.

8. Two persons start at noon from towns 60 miles apart. One walks at the rate of 4 miles an hour, but stops $2\frac{1}{2}$ hours on the way. The other walks at the rate of 3 miles an hour without stopping. When and where will they meet?

9. Divide 225 into 2 parts, so that three times the greater may exceed 7 times the less by 45.

10. Prove that, if $\frac{a}{b}=\frac{c}{d}$, $\frac{7a+5b}{8a-3b}=\frac{7c+5d}{8c-3d}$.

11. Solve the equations

(1) $\frac{1}{x}+\frac{1}{2x}-\frac{1}{3x}=\frac{7}{3}$.

(2) $\frac{3x-1}{2x-1}-\frac{4x-2}{3x-2}=\frac{1}{6}$.

(3) $\left.\begin{array}{l}\frac{2x}{3}+\frac{3y}{2}=16\frac{1}{6}\\ \frac{3x}{2}-\frac{2y}{3}=16\frac{1}{6}\end{array}\right\}$.

(4) $\left.\begin{array}{l}x-y=\frac{1}{6}\\ x^2-y^2=\frac{5}{36}\end{array}\right\}$

12. A is twice as old as B. Nine years ago he was three and a half times as old as B. Find their present ages.

XXVI.

1. If $a=2$, $b=3$, $x=6$, $y=5$, what is the value of $a+2x-\{b+y-[a-x-(b-2y)]\}$?

2. Add together

$ax-by$, $x+y$, and $(a-1)x-(b+1)y$.

3. Multiply $\frac{1}{x^2}-\frac{1}{y^2}$ by $\frac{x}{y}+\frac{y}{x}$.

Divide $\frac{a}{a-x}-\frac{a-x}{a}$ by $\frac{2ax-x^2}{(a-x)^2}$.

4. Simplify $\dfrac{a-b+\dfrac{b^2}{a+b}}{a+b+\dfrac{b^2}{a-b}}$

and $\dfrac{2}{x-2}-\dfrac{1}{x+2}-\dfrac{x+6}{x^2+4}$.

5. Reduce to the lowest terms

(1) $\dfrac{2x^2-8x+6}{x^2-5x+4}\times\dfrac{x^2-9x+20}{x^2-10x+21}\div\dfrac{2x^2-10x}{x^2-7x}$.

(2) $\dfrac{x^2+(a+b)x+ab}{x^2+(a-b)x-ab}$.

6. Find the G.C.M. of

$3x^3+x^2+4x-4$, and $6x^3-7x^2+5x-2$,

and the L.C.M. of

x^4-4x^2, $4x^3+4x^2-24x$, x^2-x-6.

7. Extract the square root of

(1) $4x^4+32x^2+96+\dfrac{64}{x^4}+\dfrac{128}{x^2}$.

(2) $a^4+\dfrac{1}{a^4}-2\left(a^2+\dfrac{1}{a^2}\right)+3$.

8. Solve the following equations:—

(1) $\frac{1}{3}(x-4)-\frac{1}{12}(8-3x)=1\frac{1}{2}$.

(2) $\dfrac{x}{x+b-a}+\dfrac{b}{x+b-c}=1$.

(3) $\dfrac{6x+13}{15}-\dfrac{3x+5}{5x-25}=\dfrac{2x}{5}$.

(4) $\left.\begin{aligned}\frac{x+y}{7}-\frac{x-y}{2}&=3\\ 2y-\frac{3x-y}{5}&=14\end{aligned}\right\}$

(5) $\left.\begin{aligned}x+y+z&=90\\ 2x+40&=3y+20\\ 2x+40&=4z+10\end{aligned}\right\}$

9. Divide 91 into two such parts that, the greater being divided by their difference, the quotient may be 7.

10. The ages of two persons are in the ratio of 3 to 4; but 10 years ago the ratio was that of 2 to 3: what are their ages?

11. When are the hands of a clock together between the hours of 6 and 7?

12. A person distributes 22*s.* in charity, giving 6*d.* to each child, 1*s.* to each woman, and 1*s.* 6*d.* to each man; there are twice as many women as men, and twice as many children as women; how many persons are relieved altogether?

13. Simplify

(1) $a^{-\frac{1}{2}}b^{\frac{2}{3}}c^{-1} \times a^{-\frac{2}{3}}b^{\frac{1}{2}}c^{-\frac{3}{2}} \div a^{-\frac{13}{6}}b^{\frac{1}{6}}c^{-\frac{1}{2}}$.

(2) $\sqrt{56} \times \sqrt{63} \times \sqrt{50}$.

XXVII.

1. If $a=1$, $b=-1$, $c=2$, find the value of

$$20ab-17bc+16ac-5(a^2+b^2+c^2).$$

2. From $(a+1)^2x-(b-1)^2y$ take $(a+1)ax-(b-1)by$.

3. Multiply $a^2x^2+abxy+b^2y^2$ by $ax-by$; and divide $a^4x^4+a^2b^2x^2y^2+b^4y^4$ by $a^2x^2+abxy+b^2y^2$.

4. Resolve into their simple factors

(1) $3a^3-12a$;

(2) a^2-a-2;

(3) $2x^2-7xy+6y^2$.

5. Find the G.C.M. of

$$20a^4+a^2-1 \text{ and } 25a^4+5a^3-a-1.$$

6. Simplify

(1) $\dfrac{2\left(x^2-\frac{1}{4}\right)}{2x+1}+\frac{1}{2}$. (2) $\dfrac{a+\dfrac{b-a}{1+ab}}{1-\dfrac{a(b-a)}{1+ab}}$.

7. Divide

$$\frac{2}{x}-\frac{1}{a+x}+\frac{1}{a-x} \text{ by } \frac{a+x}{a-x}-\frac{a-x}{a+x}.$$

8. Extract the square root of

(1) $16x^4-32x^3+24x^2-8x+1$.

(2) $9a^2x^2-12abxy+6axz+4b^2y^2-4byz+z^2$.

9. Solve the equations

(1) $\frac{1}{4}(x-3)+\frac{1}{3}(x-4)=\frac{1}{2}(x-5)+\frac{1}{8}(x+1)$.

(2) $\dfrac{(4x-5)x}{4x-1}+\dfrac{1}{3x}=x-1$.

(3) $\dfrac{a}{bx}+\dfrac{b}{ax}=a^2+b^2$.

(4) $\left.\begin{aligned}\frac{x+y}{2}-\frac{x-y}{3}&=8\\ \frac{x+y}{3}+\frac{x-y}{4}&=11\end{aligned}\right\}$

(5) $\left.\begin{aligned}(a-b)x&=(a+b)y\\ x+y&=c.\end{aligned}\right\}$

10. Find a number such that one-third of it exceeds $\frac{3}{8}$ of the remainder by 10.

11. A carriage, horse, and harness are worth together 144*l.*; the carriage is worth $\frac{4}{5}$ of the horse's value, and the harness $\frac{3}{5}$ of the difference between the values of the horse and carriage; what is the value of each?

12. In what time will a cistern which holds 670 gallons be filled by two taps running simultaneously, one of which lets in 14 gallons in five minutes and the other *5 gallons* in three minutes?

13. What fraction is that which becomes $= \frac{1}{2}$ when 2 is added to its numerator, but $= \frac{1}{3}$ when 1 is taken from its denominator?

XXVIII.

1. If $a=1$, $b=3$, $c=5$, $d=7$, find the value of
$$a-2b-[3c-d-\{3a-(5b-c-8d)\}-2b].$$

2. Simplify $(x-3)-(3-x)(1-x)-(x-3)(5\quad 2x)$.

3. From
$$(a+b)^2-2bx \text{ take } \{(a+b)(a-x)-(a-b)(b-x)\}.$$

4. Multiply $\frac{x}{y}-\frac{y}{x}$ by $xy-\frac{1}{xy}$, and divide
$$x^4-\tfrac{5}{2}x^3y+\tfrac{13}{3}x^2y^2-\tfrac{13}{6}xy^3+y^4 \text{ by } x^2-\tfrac{1}{2}xy+\tfrac{1}{3}y^2.$$

5. Find the G.C.M. of
$$2x^5-11x^2-9 \text{ and } 4x^5+11x^4+81,$$
and the L.C.M. of
$$5a^3-20ac^2,\ a^2+ac-2c^2,\ a^2+3ac+2c^2.$$

6. Simplify

(1) $$\frac{1}{2x-1}-\frac{2x-\frac{1}{2x}}{4x^2-1}+\frac{2x+\frac{1}{2x}}{4x^2+1}-\frac{1}{2x+1}.$$

(2) $$\left(\frac{a}{x^2}+\frac{x}{a^2}\right)\div\left(\frac{1}{a^2}-\frac{1}{ax}+\frac{1}{x^2}\right).$$

(3) $$\frac{\frac{a+b}{a-b}-\frac{a-b}{a+b}}{1-\frac{a^2+b^2}{(a+b)^2}}.$$

(4) $$\frac{a^{-\frac{1}{2}}b^{\frac{1}{2}}}{\frac{1}{b^{\frac{1}{2}}c^{-\frac{1}{2}}}}\times\frac{\frac{1}{b^{\frac{1}{2}}c^{\frac{1}{2}}}}{c^{\frac{1}{2}}a^{-\frac{1}{2}}}\times\frac{a^{\frac{1}{2}}c^{\frac{3}{2}}}{b^{\frac{1}{2}}c^{-\frac{1}{2}}}.$$

7. Extract the square root of

(1) $9x^4-30x^3+37x^2-20x+4$;

(2) $x^2-xy+\frac{y^2}{4}+2xz-zy+z^2$.

8. Solve the equations

(1) $\frac{x-2}{3}+\frac{x+3}{4}-\frac{x-4}{6}=7$.

(2) $\frac{x}{x+2}+\frac{4}{x+6}=1+\frac{x+1}{x+2}-\frac{x+4}{x+6}$.

(3) $\left.\begin{aligned}\frac{x+y}{12}-\frac{x-y-2}{5}&=1\\ 9y-5x&=10\end{aligned}\right\}$

(4) $\left.\begin{aligned}\frac{x}{a}-\frac{y}{b}&=1\\ \frac{x}{b}+\frac{y}{a}&=\frac{a}{b}\end{aligned}\right\}$ (5) $\left.\begin{aligned}\frac{x}{2}+\frac{y}{3}=\frac{y}{2}+\frac{z}{8}&=4\\ \frac{z}{16}-\frac{x}{12}&=\frac{1}{6}\end{aligned}\right\}$

9. Find a number which is as much greater than 63 as its half is less than 93.

10. A and B have the same income; A puts by $\frac{1}{8}$ of his, but B, by spending every year 80*l.* more than A, finds himself at the end of three years 20*l.* in debt. What is the income of each?

11. A person swimming in a stream which runs $1\frac{1}{2}$ mile per hour finds that it takes him four times as long to swim a mile up the stream as it does to swim the same distance down. At what rate does he swim?

12. The wages of 10 men and 8 boys for a day amount to 1*l.* 17*s.*; if four men together receive one shilling more than six boys, what are the wages of each man *and boy*?

XXIX.*

1. Define a^m, where m is a positive integer. Prove that $(a^4)^3=(a^3)^4$, $a^{\frac{1}{3}}=\sqrt[3]{a}$.

2. Divide

$a+b-c-3(b^{\frac{1}{3}}-c^{\frac{1}{3}})(c^{\frac{1}{3}}-a^{\frac{1}{3}})(a^{\frac{1}{3}}+b^{\frac{1}{3}})$ by $a^{\frac{1}{3}}+b^{\frac{1}{3}}-c^{\frac{1}{3}}$,

and $x^4+x^{-4}+4(x^2+x^{-2})+6$ by $x^2+x^{-2}+2$.

3. Reduce to their simplest forms

(1) $\dfrac{1}{(2x-1)^2}+\dfrac{1}{(2x+1)(1-2x)}+\dfrac{1}{2x^2(2x-1)(1-4x^2)}$.

(2) $\sqrt[3]{\dfrac{x^2+y^2-\dfrac{4x^2y^2}{x^2+y^2}}{x^2-y^2+\dfrac{4x^2y^2}{x^2-y^2}}+\dfrac{2}{x^2\left(\dfrac{1}{x^2}+\dfrac{1}{y^2}\right)}}$

4. Extract the square root of

(1) $81a^{-\frac{5}{3}}b^{\frac{2}{5}}c$. (2) $\dfrac{4x^2}{a^2}+\dfrac{a^2}{4x^2}-\dfrac{4x}{a}-\dfrac{a}{x}+3$.

5. Find (1) the G.C.M., (2) the L.C.M. of the three quantities

$$12xy(x^5y-2x^3y^3+xy^5),\ 20(x^8-2x^5y^3+x^2y^6),\text{ and }8y(x^7-xy^6);$$

and (3) the L.C.M. of $b^{\frac{1}{2}}c^{\frac{2}{3}}$, $c^{\frac{1}{2}}a^{\frac{2}{3}}$, $ab^{\frac{1}{4}}$, and $a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{1}{4}}$.

6. Express in their simplest forms

(1) $\frac{1}{6}(\sqrt[3]{3}+\frac{1}{2}\sqrt[3]{192}+\sqrt[3]{81})$; (2) $\sqrt[3]{36}\times\sqrt[3]{48}$.

(3) $\left[\dfrac{x^{\frac{5}{2}}y^{\frac{4}{3}}}{z^{-\frac{5}{4}}}\times\dfrac{z^4}{x^{-3}y^{-\frac{5}{3}}}\div\dfrac{y^{-2}z^{\frac{1}{4}}}{x^{-\frac{1}{2}}}\right]^{\frac{1}{5}}$; (4) $\{(a^{-\frac{1}{3}})^{\frac{3}{2}}\}^{-\frac{2}{4}}$.

* This and the papers which follow include Quadratic Equations and Proportion.

7. Solve the equations

(1) $\frac{1}{3}\cdot\frac{x+1}{x+2}-\frac{1}{4}\cdot\frac{x+2}{x+3}=\frac{x+4}{12x}$.

(2) $9x^2=18x+1$. (3) $x+\sqrt{x}=1\frac{1}{5}$.

(4) $\left.\begin{matrix}x^2-4y^2=9\\ x+2y=9\end{matrix}\right\}$ (5) $\left.\begin{matrix}x+y=5\\ x^2-3xy+4y^2=7\end{matrix}\right\}$

8. If α and β are the roots of $x^2-px+q=0$, (1) shew that the product of $x-\alpha$ and $x-\beta$ produces the expression x^2-px+q, and (2) shew that

$$\frac{\alpha}{\beta}-\frac{\beta}{\alpha}=\frac{p}{q}\sqrt{p^2-4q}.$$

9. If $a:b::c:d::e:f$, shew that

(1) $ad=bc$, and (2) $a:b::a+c+e:b+d+f$.

10. Two navvies A and B working together can dig out 20 cubic yards of earth in a day. They work together for 6 days, after which A leaves off work and B continues for 4 days more. It is then found that they have dug out 156 cubic yards. How much can each dig in a day?

XXX.

1. Find the value of

$\frac{x^3-y^3}{x-y}$, when $x=2^{\frac{1}{2}}a^{\frac{1}{2}}$, and $y=2^{-\frac{1}{2}}a^{\frac{1}{2}}$.

2. Shew that if $2s=a+b+c$, then

$16s(s-a)(s-b)(s-c)=2b^2c^2+2c^2a^2+2a^2b^2-a^4-b^4-c^4$.

3. Multiply

$a\sqrt{1-b^2}+b\sqrt{1-a^2}$ by $a\sqrt{1-b^2}-b\sqrt{1-a^2}$;

and divide $x^{\frac{1}{2}}-y^{\frac{1}{2}}$ by $x^{\frac{1}{4}}-y^{\frac{1}{10}}$.

Explain the meaning of a fractional index.

4. Reduce to their simplest forms

(1) $\dfrac{-\frac{1}{m}}{\frac{1}{m}-\dfrac{1}{m-n+\frac{n^2}{m+n}}}$ (2) $\dfrac{1-\frac{m-1}{m}}{1+\frac{1}{m-1}} \div \dfrac{m-1}{m}$.

(3) $\dfrac{x^2+y^2+z^2+2yz+2zx+2xy}{x^2-y^2-z^2-2yz}$.

(4) $\dfrac{1}{4(x-1)(x+2)}+\dfrac{1}{6(1-x^2)}+\dfrac{1}{4(x+1)(x-2)}-\dfrac{3}{2(4-x^2)}$.

5. Prove that

$$\frac{x^2-y^2+(x-y)\sqrt{xy}}{\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}}=x^2\sqrt{y}-y^2\sqrt{x}.$$

6. Find the G.C.M. of
$4m^2n^2(m^2+mn+n^2)^2$, $6mn(m^6-n^6)$, and $18m^3(m^3-n^3)^2$.
Find also the L.C.M. of the same three quantities.

7. Find the square root of

$$9x^2+6y-\frac{4}{x}+\frac{4}{y^2}-\frac{12x}{y}+\frac{y^2}{x^2},$$

and the cube root of $125a^{-\frac{1}{2}}b^6c^{-7}$; and raise $\dfrac{32a^{-10}x^{-\frac{1}{2}}}{b^3y^{\frac{5}{3}}}$ to the power of $-\frac{4}{5}$.

8. Solve the equations

(1) $\dfrac{1}{abx}+\dfrac{ab}{x}-\dfrac{1}{a^2}=b^2$. (2) $x+\sqrt{x^2-\sqrt{1-x^2}}=1$.

(3) $x^2+\frac{5}{2}x-\frac{1}{2}=1$. (4) $\left.\begin{matrix} ax-by=0 \\ x+y=c \end{matrix}\right\}$.

(5) $2(x+y^{-1})=3(x^{-1}-y)=4$.

9. Prove that if $a : b :: \alpha : \beta$, then

$$a+b : a-b :: a+\beta : a-\beta.$$

10. The circumference of the fore-wheel of a carriage is 9 feet, and that of the hind-wheel 12 feet. What is the distance gone over when the fore-wheel has made two more revolutions than the hind-wheel?

XXXI.

1. Find the value of

$$x^2+xy+y^2, \text{ if } x=1+\sqrt{2},\ y=1-\sqrt{2}.$$

2. Find the L.C.M. of

(1) $x^2-1,\ x^2-x-2,\ x^2+x-2.$

(2) $2xy^2-2x^3,\ x^3+y^3,\ 3x^2(x-y)^2.$

3. Simplify

(1) $$\frac{\frac{x}{y}+\frac{y}{x}-1}{\frac{x^2}{y^2}+\frac{x}{y}+1} \times \frac{x-y}{1+\frac{y}{x}} \div \frac{1+\frac{y^3}{x^3}}{\frac{x^2}{y}-\frac{y^2}{x}}.$$

(2) $$\frac{y}{2x^2-2xy}-\frac{4x+5y}{4(x^2-y^2)}+\frac{4x-5y}{4(x-y)^2}.$$

4. Express $\sqrt{x}$ and $\sqrt[3]{x\sqrt{y}}$ with fractional indices; and simplify

$$(a^{-m}a^{\frac{1}{n}})^n \text{ and } \left(\frac{ay}{x^2}\right)^{\frac{1}{2}} \times \left(\frac{bx^2}{y^2}\right)^{\frac{1}{3}} \div \left(\frac{b^2a^3x^2}{y^3}\right)^{\frac{1}{6}}.$$

5. Find the square root of

(1) $x^4+\frac{3}{2}x^3+4\frac{1}{16}x^2+\frac{3}{2}x+1.$

(2) $x^{2(m+n)}+x^{2(m-n)}-4x^m(x^{m+n}+x^{m-n})+6x^{2m}.$

6. Solve the equations

(1) $bx+ay=2ab,\ a^2x+b^2y=a^3+b^3.$

(2) $4x^2+13x-12=0.$

(3) $\dfrac{\sqrt{x}+2a}{\sqrt{x}+2b}=\dfrac{\sqrt{x}+4a}{\sqrt{x}+3b}.$

(4) $x+\sqrt{5x+10}=8.$

7. Given the quadratic equation $ax^2+bx+c=0$, express (1) the sum of the roots, and (2) the difference of the roots in terms of a, b, c.

What is the condition that the roots shall be equal but of opposite signs?

8. If $a : b :: c : d$, shew that

(1) $(a-c)-(b-a)=\dfrac{(a-b)(a-c)}{a}.$

(2) $(a^2+c^2)^3 : (b^2+d^2)^3 :: (a^3+c^3)^2 : (b^3+d^3)^2.$

9. Find x and y from the proportions

$$2x+y : y :: 3y : 2y-x,$$
$$2x+1 : y :: 2x+6 : y+2.$$

10. A pays on his income an income-tax of 3*d.* in the pound, and a poor-rate exceeding what he pays for income-tax by 29*l.* 10*s.*, and he has 750*l.* 10*s.* left. Find his income.

XXXII.

1. Mutiply together

$$x+\sqrt{xy}+y,\ x-\sqrt{xy}+y,\ x^2-xy+y^2;$$

and find the cube of $x^{\frac{1}{3}}+1-x^{-\frac{1}{3}}.$

2. Divide

$(1-x^2)(1-y^2)(1-z^2)-(x+yz)(y+zx)(z+xy)$

by $1-x^2-y^2-z^2-2xyz,$

and $a^2-b^2+c^2-d^2+2bd-2ac$ by $a+b-c-d.$

3. Find the G.C.M. of $3x^4-4x^3+1$ and $4x^4-5x^3-x^2+x+1$, and the L.C.M. of a^2+b^2, a^2-b^2, $a^3+a^2b+ab^2+b^3$, $a^3-a^2b+ab^2-b^3$.

4. Simplify

$$\left\{\frac{c-b}{(a-b)\,(a-c)}+\frac{a-c}{(b-c)\,(b-a)}+\frac{b-a}{(c-a)\,(c-b)}\right\} \div\frac{2(a^2+b^2+c^2-bc-ca-ab)}{(a-b)\,(b-c)\,(c-a)}.$$

5. Solve the equations

(1) $\sqrt{x}+\sqrt{4+x}=\dfrac{4}{\sqrt{x}}$. (2) $\dfrac{x+1}{y}=\dfrac{y+7}{x}=2$.

(3) $\dfrac{b^2-c^2}{x^2+a^2}+\dfrac{c^2-a^2}{x^2+b^2}+\dfrac{a^2-b^2}{x^2+c^2}=\dfrac{x^6}{(x^2+a^2)\,(x^2+b^2)\,(x^2+c^2)}$.

6. If α, β are the roots of the equation $x^2-px+q=0$, prove that $\alpha+\beta=p$ and $\alpha\beta=q$.

Form and solve the quadratic equation, the product of whose roots is 42 and their sum 13.

7. Solve the equations

(1) $2x^2-21x+55=0$. (2) $x^6-20x^3=189$.

(3) $\left.\begin{matrix}2x^2+3xy=26\\3y^2+2xy=39\end{matrix}\right\}$ (4) $\left.\begin{matrix}x^3+y^3=9\\x+y=3\end{matrix}\right\}$

8. Prove that a ratio of greater inequality is diminished, and a ratio of less inequality is increased, by adding the same number to each of its terms.

If $a+b : b :: c+d : d$, then $a : a-b :: c : c-d$; and if $a : b :: b : c :: c : d$,

(1) $\dfrac{a+b}{a-b}=\dfrac{b+c}{b-c}=\dfrac{c+d}{c-d}$. (2) $a : d :: a^3 : b^3$.

9. A number having two digits is to the number formed by inverting the digits as 7 to 4; and the sum of the two numbers is 132. Find them.

XXXIII.

1. Find the value of

$$\frac{xy-a^2}{\frac{x}{y}-\frac{1}{ab}}-\frac{(x+y)(a-b)}{x^2-b^2}$$

when $x=0, y=-1, a=\frac{1}{2}, b=-\frac{1}{2}$.

2. Multiply $x^2-xy-xz$ by $xy+y^2-yz$; and find the remainder when x^6+1 is divided by x^2+x+1.

3. Find the G.C.M. and the L.C.M. of

(1) $34a^2bx^3y^4$, $27x^5y^3b^3$, $51y^2a^3x$;

(2) $6x^3+6x^2-12x$, $4x^4-4x$, $3x^3+3x^2+3x$.

4. Find the G.C.M. of

$3x^3+4x^2+2x+21$ and $9x^4-37x^2+22x-14$.

5. Simplify

(1) $\dfrac{x^3-8y^3-z^3-6xyz}{x^3-x^2z-2x^2y}$;

(2) $\dfrac{x-1}{(x-2)(x-3)}+\dfrac{2(x-2)}{(x-3)(1-x)}-\dfrac{3-x}{(x-2)(x-1)}$;

(3) $\dfrac{x}{y+\dfrac{x^2}{x+y}} \div \dfrac{\dfrac{x}{y}-\dfrac{y}{x}}{\dfrac{x^2}{y}-\dfrac{y^2}{x}}$.

6. Write down the cube, and also the cube root, of $8a^{-\frac{1}{3}}b^{-\frac{2}{3}}c$; and simplify

$$\sqrt{a^{\frac{4}{3}}(b^{\frac{1}{3}}a)^{-\frac{1}{2}}} \times \sqrt[3]{(a^{-3})^{\frac{1}{2}}b^{\frac{1}{2}}}.$$

7. Extract the square root of

$$x^4+\frac{1}{9}+2x+\frac{1}{x^2}-\frac{2}{3}x^2-\frac{2}{3x}.$$

8. Solve the equation $ax^2+2bx+c=0$, and shew that the two roots are equal if b is a mean proportional between a and c.

Form the quadratic equation whose roots are 1 and -2.

9. Solve the equations:—

(1) $\dfrac{x+b}{x+2a}+\dfrac{3a}{3x+4a+2b}=1.$

(2) $\left.\begin{aligned}\dfrac{x+2y}{2}+\dfrac{x-2y}{3}&=\dfrac{1}{12},\\ \dfrac{y+x}{y-x}&=\dfrac{1}{3};\end{aligned}\right\}$

(3) $3x^2-x-2=0;$ (4) $x-\dfrac{1}{x}=2.$

(5) $\left.\begin{aligned}x^2-xy&=\tfrac{3}{2}\\ xy+y^2&=1\end{aligned}\right\}.$

10. When are four quantities said to be proportional?

If $a:b::b:c$, then will $a^2+2b^2 : b^2+2c^2 :: a : c$; and if $a+b : a-b :: c+d : c-d$, then a, b, c, d are proportionals.

11. A and B together can do a certain piece of work in 6 days, while A by himself would finish it nine days sooner than B by himself; in what time would each do it by himself?

12. Three persons divide a certain sum of money between them in the following manner: A takes the nth part of the whole, together with $\frac{a}{n}$£; B takes the nth part of the remainder, together with $\frac{a}{n}$£; C takes the nth part of what still remains, together with $\frac{a}{n}$£; after *ch nothing* remains. Find the sum of money.

XXXIV.

1. Find the value, when $a=1$, $b=-\frac{1}{2}$, $c=0$, of the expression

$$\frac{a-[b-c-\{2a-2b-\frac{1}{2}(3c-b)\}]}{a-\dfrac{b}{c-\dfrac{1}{a}}}.$$

2. Multiply x^2-6x+9 by x^2+3x+9, and find the remainder when $x^3-8y^3-27z^3$ is divided by $x-2y-3z$.

3. Find the L.C.M. of

(1) $8(a^4b^3+a^2b^5)$, $36(a^4b^3-b^7)$, $27(a^2b^3-b^5)$;

(2) $8x^2-12x-8$, $8x^2-6x+1$, $8x^3-2x$.

4. Find the G.C.M. of

x^4+x^3-3x-9 and $2x^5-6x^3-x^2+3$.

5. Simplify:

(1) $\dfrac{(y^2-zx)^2-(z^2-xy)(x^2-yz)}{(z^2-xy)^2-(x^2-yz)(y^2-zx)}$;

(2) $\dfrac{\frac{1}{x^2}+\frac{1}{xy}+\frac{1}{y^2}}{x^2-xy+y^2}\times\dfrac{x^3+y^3}{\frac{1}{x^3}-\frac{1}{y^3}}\div\dfrac{x-y+\frac{4xy}{x-y}}{x+y}$.

(3) $\dfrac{1}{3a(2a+b)+3b(2b+a)}-\dfrac{1}{6ab-3a^2-3b^2}+\dfrac{(a+b)}{2(a^3-b^3)}$.

6. Find the square root of

$$\frac{1}{4}-4xy+\frac{x^2}{y}-2\frac{y^2}{x}+4\frac{y^4}{x^2}+\frac{x^4}{y^2}.$$

7. Divide $x^{\frac{4}{3}}+x^{\frac{2}{3}}y^{\frac{1}{2}}+y$ by $x^{\frac{2}{3}}+x^{\frac{1}{3}}y^{\frac{1}{2}}+y^{\frac{1}{2}}$; and simplify:

(1) $\left\{\frac{a^{-2}\,b^{-\frac{1}{3}}}{\sqrt[3]{ab}}\sqrt{\frac{a^7}{a^{-1}\,b^{-\frac{5}{3}}}+7a^{\frac{3}{2}}}\right\}^{\frac{1}{3}}$;

(2) $\frac{1}{\left\{\sqrt{p^2-1}+\sqrt{p^2+1}\right\}^2}+\frac{1}{\left\{\sqrt{p^2-1}-\sqrt{p^2+1}\right\}^2}$

8. Solve the equation $x^2+px+q=0$.

Shew that the equation $x^2+x\,(2q-p^2)+q^2=0$ is such that each of its roots is the square of one of the roots of the equation $x^2+px+q=0$.

9. Solve the equations:

(1) $\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{x+a+b}+\frac{1}{x}$;

(2) $\left.\begin{aligned}\frac{21-5x}{5}+\frac{27-y}{2}&=\frac{11}{5}+8x,\\ \frac{3}{y}-\frac{5}{x}&=\frac{31}{xy};\end{aligned}\right\}$

(3) $2x^2-8x+7=0$; (4) $8x^2=3xy^2=72y$.

10. When are two quantities said to have the duplicate ratio of two others?

What quantity must be subtracted from each term of the ratio $a:b$ to obtain the duplicate ratio of a to b?

If $a^2+b^2 : b^2+c^2 :: a : c$, prove that b is the mean proportional between a and c.

11. 200 people attended a concert, for admission to which two prices, differing from one another by 3*s.*, were charged. 12*l.* 10*s.* was taken at the higher price, and 15*l.* at the lower. Find the prices of the tickets.

12. By selling a horse for 24*l.* I lost as much per cent. *as the horse* cost me. Shew that the horse may have *cost either of* two sums, and find them.

XXXV.

1. If $a=1$, $b=2$, $c=-3$, $d=0$, find the value of

$$\frac{a^3b+a^2c+b^3c-c^3d}{a^2d+b^2-c^2}-\sqrt{\frac{4bc-ac-ab^2}{a-b+cd}}.$$

2. Multiply

$$8a^3-4a^2+2a+1 \text{ by } 1-2a,$$

and divide

$$2x^3+2ax^2+3bx^2+2bx+3abx+3b^2 \text{ by } 2x+3b.$$

3. Find the L.C.M. of

$$24(a^2-b^2),\ 30(a^3-b^3),\ 32(a^3+b^3),\ 160(a^3+2a^2b+2ab^2+b^3).$$

4. Find the G.C.M. of

$$x^4+7x^3-3x-5 \text{ and } x^4-9x^2+2x+6.$$

5. Simplify

(1) $\dfrac{3}{x^2+x-2}-\dfrac{5}{x^2-x-6}+\dfrac{2}{x^2-4x+3}$;

(2) $\dfrac{x+\frac{1}{x}+2}{x-\frac{1}{x^2}}\times\dfrac{1-\frac{2}{x}+\frac{1}{x^2}}{x^2+\frac{1}{x}}\div\dfrac{\frac{1}{x}-x}{\left(\frac{1}{x}+x\right)\left(x^2+\frac{1}{1+x^2}\right)}$;

(3) $(a^{-2}b^{-\frac{3}{2}}\sqrt{a^{-1}b^3}+ab^{\frac{1}{2}}\sqrt{a^{-7}b^{-1}})\ \sqrt{a^5b^2}$.

6. Extract the square root of

$$\frac{4a^2}{b^2}+\frac{b^2}{4a^2}-8\frac{a}{b}+2\frac{b}{a}+2.$$

7. Divide

$a-b$ by $a^{\frac{1}{3}}-b^{\frac{1}{3}}$, and $a+b$ by $a^{\frac{1}{3}}+b^{\frac{1}{3}}-\sqrt{2}a^{\frac{1}{3}}b^{\frac{1}{3}}$.

8. Shew that the equation

$$x^3+px+q=0$$

can have no more than two roots, and that the sum of the cubes of the roots is $3pq-p^3$.

9. Solve the equations

(1) $\dfrac{a+b}{x-a-b}-\dfrac{a}{x-a}-\dfrac{b}{x-b}=0$;

(2) $\sqrt{x-2}+\sqrt{x+3}=\sqrt{4x+1}$;

(3) $\dfrac{x-1}{x+1}-\dfrac{x-2}{x+2}=\dfrac{8x-5}{(2x+1)^2}$;

(4) $\left.\begin{array}{c} \dfrac{3-x}{2}-\dfrac{y-4}{3}=\dfrac{x}{3}+\dfrac{y}{6} \\ 3x-4y=16 \end{array}\right\}$

10. Prove that, if $a : b :: c : d$, then

(1) $a+b : a-b :: c+d : c-d$,

(2) $ac : bd :: a^2+2c^2 : b^2+2d^2$.

11. A and B play at cards: when they begin, A has three times as much money as B; after winning $2l.$, A finds that he has seven times as much as B. With what sum did each begin?

12. There are two rectangular grass plots. The length of one is to its breadth as 4 : 3. The breadth of the other exceeds by 9 yards the breadth of the former, and half its length exceeds by 3 yards the length of the former. If it cost $1l.$ $16s.$ to turf the former, and $9l.$ to turf the latter at the same rate, find the dimensions of the grass plots, and the cost of turfing per square yard.

13. Solve the equations:

(1) $\left.\begin{array}{l} x^2+y^2=394 \\ x-y\ =\ 2 \end{array}\right\}$

(2) $\left.\begin{array}{l} x^2+xy=a^2+ab \\ y^2+xy=b^2+ab \end{array}\right\}$

XXXVI.

1. Prove the rule for subtracting one algebraical expression from another;

and take $(p-q)^2 x^3-3pqx^2-(p+q)^2 x-q^3$

from $(p^2+q^2)(x^3-x)-7pqx^2-q^3$.

2. Multiply $\frac{x^2}{2}-\frac{xy}{3}+y^2$ by $\frac{x^2}{2}+\frac{xy}{3}+y^2$, and divide $1+4a^4b^4$ by $2a^2b^2-2ab+1$.

3. If a quantity c be a common measure of a and b, it will also measure the sum or difference of any multiples of a and b.

Find the G.C.M.

(1) of $3x^4-x^2y^2-2y^4$
and $10x^4+15x^3y-10x^2y^2-15xy^3$;

and (2) of $(a+2c)(a-c)-b(4a-3b-c)$
and $(a+2b)(a-b)-c(4a-b-3c)$.

4. Simplify the expressions:

(1) $\frac{1}{x+2}+\frac{2}{x^2-4}-\frac{x^2+4}{x^3+8}$,

(2) $\frac{3b}{3ax-5by}-\frac{1}{\frac{ax}{b}-y-\frac{2by^2}{3ax-2by}}$.

5. Solve the equations:

(1) $\frac{x-7}{11}-\frac{3x-5}{7}+\frac{125}{77}=2x-17$.

(2) $\frac{x+1}{x-1}-\frac{x-9}{x-7}=\frac{x}{x-2}-\frac{8-x}{6-x}$.

(3) $$x-\frac{3x-2y}{7}=y-\frac{4x-5y}{19},$$
$$y-\frac{2x-7y}{6}=2(x-1)-\frac{3x-7}{8}.$$

6. The diameter of one coin is 1·3 inches, and of another 1·2 inches; 160 placed in contact in a straight line measure a pole: how many are there of each kind?

7. Shew that $\sqrt[3]{40}-3\sqrt[3]{320}+4\sqrt[3]{135}=2\sqrt[3]{5}$; and find the square root of $69-7\sqrt{80}$.

8. Solve the equations:

(1) $$x-15\tfrac{3}{4}+\frac{5}{x-15\frac{3}{4}}=6.$$

(2) $$\left.\begin{aligned}x^2-xy&=8x+3\\xy-y^2&=8y-6\end{aligned}\right\}.$$

9. Define *ratio* and *proportion*.

If x, y, z be proportional to given quantities a, b, c, respectively, and if the sum of their squares be 100, find their values.

10. Shew that a ratio of greater inequality is diminished by adding the same quantity to both terms of the ratio.

If $a : b :: b : c$, then $\frac{1}{a^2-b^2}+\frac{1}{c^2-b^2}+\frac{1}{b^2}=0$.

If α, β be the roots of the equation

$$x^2-px+q=0,$$

then
$$\alpha+\beta=p,\ \alpha\beta=q.$$

If the same value of x satisfies the equations $ax^2+bx+c=0$, and $cx^2+bx+a=0$, then $a\pm b+c=0$.

12. In a right-angled triangle, if the sum of the sides which contain the right angle be to the hypotenuse in the ratio of $m : n$, prove that m cannot be greater than $n\sqrt{2}$.

XXXVII.

1. Find the value of

$$\sqrt{\frac{(\sqrt{x^2+y^2}+xy)^2-1}{2}}$$

when $x=3$, $y=4$; and of

$$\sqrt[3]{6abc+(b+c)^3+(c+a)^3+(a+b)^3-(a+b+c)^3}$$

when $a=5$, $b=4$, $c=3$.

2. Divide

a^3-b^3 by $a-b$, and $x^2+y^2+1-2y+2x-2xy$ by $x-y+1$.

3. Prove that $a^m a^n=a^{m+n}$, when m and n are positive integers; and state the steps by which an interpretation is assigned to a^m when m is fractional or negative.

4. Find the G.C.M. of

$x^3-9x^2+26x-24$ and $x^3-12x^2+47x-60$.

5. Reduce to its simplest form the expression

$$\frac{\dfrac{x+a}{x+b}-\dfrac{x+b}{x+a}}{\dfrac{x-b}{x-a}-\dfrac{x-a}{x-b}}$$

and prove that $\dfrac{b-c}{a}+\dfrac{c-a}{b}+\dfrac{a-b}{c}=\dfrac{(b-c)(c-a)(a-b)}{abc}$.

6. If

$$ab(c+d-e-f)+cd(e+f-a-b)+ef(a+b-c-d)=0,$$

prove that

$$(a-c)(b-f)(e-d)=(a-e)(b-d)(c-f).$$

7. Solve the equations:

(1) $\dfrac{10x+3}{7x-1}+\dfrac{4x+11}{5x-8}=2\frac{8}{35}$,

(2) $\sqrt{\frac{x}{a}}+\sqrt{\frac{a}{x}}=\sqrt{\frac{x}{b}}+\sqrt{\frac{b}{x}}$,

(3) $7x+10y=149, \quad 11x-14y+119=0.$

8. The united ages of a man and his wife are six times the united ages of their children. Two years ago their united ages were ten times the united ages of their children, and six years hence their united ages will be three times the united ages of the children. How many children have they?

9. Solve the equations

(1) $(x-1)(x-2)(x-3)=(x-5)(x-6)(x-7).$

(2) $\begin{cases} x^2+y^2+(1+xy)^2=14; \\ xy(x+y)=6. \end{cases}$

What two numbers are those whose difference is 18, and the sum of whose squares is 554?

10. Prove that the ratio $\frac{ma+nc+pe+\&c.}{mb+nd+pf+\&c.}$ is equal to each of the ratios $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$ &c. when these ratios are all equal to each other, and that it is intermediate between the greatest and least of these ratios when they are unequal.

11. Prove that the equation

$$2(x^2-ax+a^2)(y^2-by+b^2)=x^2y^2+a^2b^2$$

is equivalent to the equation

$$(x-a)^2(y-b)^2+(bx-ay)^2=0,$$

and therefore that the only real solutions are $x=a, y=b$.

12. Prove that every quadratic equation has two roots, and no more.

If α *and* β be the roots of $x^2+bx+c=0$, prove that

$$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{b}{c}=0.$$

QUESTIONS ON EUCLID.

BOOKS I., II., III., IV.

BOOK I.

1. Define a point, a line, a straight line, a superficies, a plane superficies, a plane angle, a plane rectilineal angle, a right angle, a perpendicular, an obtuse angle, an acute angle, a term or boundary, a figure, a circle, the centre of a circle, a diameter of a circle, a semicircle, a segment of a circle, rectilineal figures, triangles, quadrilateral figures, polygons, an equilateral triangle, an isosceles triangle, a scalene triangle, a right-angled triangle, an obtuse-angled triangle, an acute-angled triangle, a square, an oblong, a rhombus, a rhomboid, a trapezium, parallel straight lines, a parallelogram, the diameter of a parallelogram.

2. Of what kind are the boundaries (1) of a line, (2) of a superficies?

3. What is the distinction between a plane angle and a plane rectilineal angle as defined by Euclid?

4. How do you know a right angle?

5. How are triangles classified (1) according to the equality or inequality of their sides, (2) according to the magnitude of their angles?

6. What is meant by the base of a triangle, and what by the hypotenuse of a right-angled triangle?

7. Denominate the different kinds of quadrilateral figures: how are they distinguished from one another?

8. Would it be a correct description of a trapezium to say that it is

(1) any quadrilateral figure?
(2) any irregular quadrilateral?
(3) any quadrilateral which is not a parallelogram?

9. What is meant by the diagonals of a quadrilateral figure?

10. Write out the Postulates.

11. What is the use of the Postulates?

12. Write out the Axioms.

13. Which of the Axioms are geometrical, i.e. have reference to geometrical magnitudes exclusively?

14. Which Axiom supplies a geometrical criterion of equality?

15. State the objections to the twelfth Axiom being really considered an axiom.

16. Explain the following terms: Definition, postulate, axiom, proposition, problem; theorem, enunciation, hypothesis, demonstration, corollary, reductio ad absurdum, à fortiori, Q.E.D., Q.E.F.

17. How are postulates and axioms related to problems and theorems?

18 Explain the meaning and importance of the words in italics in the following enunciations:—

(1) Describe an equilateral triangle upon a given *finite* straight line.

(2) Upon the same base *and on the same side of it* there cannot be two triangles having their sides which are terminated in one extremity of the base equal to one another, *and likewise* those which are terminated in the other extremity equal.

(3) If a side of a triangle be produced, the exterior angle shall be greater than either of the interior *and opposite* angles.

(4) Make a triangle the sides of which shall be equal to three given straight lines, *of which any two are together greater than the third.*

(5) If two triangles have two angles of the one equal to two angles of the other, each to each, and have one side equal to one side, *viz. either the sides adjacent to the equal angles in each, or sides opposite to them;* then shall the other sides be equal, each to each, and also the third angle of the one to the third angle of the other.

(6) The straight lines which join the extremities of equal and parallel straight lines *towards the same parts* are themselves equal and parallel.

(7) Equal triangles upon equal bases, *in the same straight line* and *towards the same parts*, are between the same parallels.

(8) The complements of the parallelograms which are *about the diameter* of any parallelogram are equal to one another.

(9) *To a given straight line apply* a parallelogram equal to a given triangle and having an angle equal to a given rectilineal angle.

19. Enunciate the Propositions from which the following Corollaries are deduced :—

(1) Every equilateral triangle is also equiangular.

(2) Every equiangular triangle is also equilateral.

(3) If two straight lines cut one another, the angles which they make with one another at the point of section are equal to four right angles.

(4) All the angles made by any number of straight lines meeting in a point are together equal to four right angles.

(5) All the interior angles of any rectilineal figure together with four right angles are equal to twice as many right angles as the figure has sides.

(6) All the exterior angles of any rectilineal figure are together equal to four right angles.

(7) Every parallelogram which has one right angle has all its angles right angles.

20. Prove the above corollaries.

21. Shew how to apply to a given straight line a parallelogram which shall be equal to a given rectilineal *figure* and have an angle equal to a given rectilineal *angle.*

22. How is it attempted to be shewn in Simpson's Euclid that two straight lines cannot have a common segment?

In what respects is the demonstration there given unsatisfactory?

In which of the previous propositions of Euclid has the truth of this property of straight lines been assumed?

23. What does Euclid mean by the equality of two triangles? and under what conditions, besides those mentioned in Prop. 4, Bk. I., does he prove two triangles to be equal?

24. What is the least number of sides which a rectilineal figure can have? Prove your answer.

25. If one side of a triangle be produced, the exterior angle is greater than either of the interior opposite angles. By what later proposition in the first book is this superseded? Shew that this is a necessary step in the reasoning.

26. How many of the exterior angles of every triangle must be obtuse? Prove your answer.

27. Construct a square which shall be equal (1) to the sum, (2) to the difference, of two given squares.

28. What properties of triangles and parallelograms respectively does Euclid prove in Bk. I.?

29. Enunciate the propositions which are proved *ex absurdo*, that is, by shewing the contrary to lead to an absurdity.

BOOK II.

1. Define a rectangle and a gnomon.

2. By what lines is a rectangle said to be contained? and by what is it contained?

3. Is the term rectangle synonymous with oblong? If not, how do they differ from one another?

4. Enunciate the Propositions from which the following Corollaries are derived:—

(1) Parallelograms about the diameter of a square are themselves also squares.

(2) The difference of the squares of two unequal lines is equal to the rectangle contained by their sum and difference.

5. Prove these corollaries.

6. Shew that Props. 2 and 3 of Bk. II. are only particular cases of Prop. 1.

7. If a straight line is divided into any two parts, the squares of the whole line and of one of the parts are equal to twice the rectangle contained by the whole and that part together with the square of the other part: where is this proposition employed?

8. Derive Prop. 8 from Props. 4 and 7.

9. Shew that the second case of Prop. 13 is included in the first case.

10. Give algebraical proofs of Props. 1–10.

BOOK III.

1. Define equal circles, a segment of a circle, the angle of a segment, an angle in a segment, a sector, and similar segments of a circle. When is a straight line said to touch a circle, circles to touch one another, straight lines to be equally distant from the centre, one straight line to be farther than another from the centre? Upon what is an angle in a circle said to insist or stand?

2. Enunciate the Propositions from which the following Corollaries are derived:—

(1) If in a circle a straight line bisects another at right angles, the centre of the circle is in the line which bisects the other.

(2) The straight line which is drawn at right angles to the diameter of a circle at its extremity touches the circle.

(3) A straight line, which touches a circle, touches it only in one point.

(4) There can be but one straight line which touches a circle in the same point.

(5) If one angle of a triangle be equal to the other two, it is a right angle.

(6) If from a point without a circle there be drawn two straight lines cutting it, the rectangles contained by the whole lines and the parts without the circle are equal to one another

3. Prove the corollaries (5), (6), in Question 2.

4. A straight line cannot cut a circle in more than two points. From which proposition does this follow?

5. In what case can two chords in a circle bisect one another?

6. Distinguish between the concave and convex part of the circumference of a circle with respect to a point.

7. How many straight lines can be drawn from the same point to touch a circle?

8. In what propositions are the following statements assumed as axioms?

(1) The sum of the doubles of two magnitudes is double the sum of the magnitudes themselves.

(2) The difference of the doubles of two magnitudes is double the difference of the magnitudes themselves?

9. Why can there not be more than one circle whose circumference passes through the angles of a given triangle?

10. What condition must be fulfilled in order that the same circumference may pass through *four* given points? or five? or any number?

11. Combine Proposition 35 and the Corollary of 36 in a single enunciation.

12. Classify the properties of the circle demonstrated in this Book under the following heads, viz. those which relate (1) to the centre, (2) to similar segments, (3) to the rectangles contained by the segments of lines which cut the circle, (4) to lines which touch the circle.

BOOK IV.

1. Describe precisely what is meant when it is said that a rectilineal figure is inscribed in another rectilineal figure, a rectilineal figure is described about another rectilineal figure, a rectilineal figure is inscribed in a circle, a rectilineal figure is described about a circle, a circle is inscribed in a rectilineal figure, a circle is described about a rectilineal figure, a straight line is placed in a circle.

2. Bracket in pairs those of the above descriptions which may be exhibited by the same figure.

3. Define a pentagon, hexagon, quindecagon; and give the derivations of these words. What are the corresponding names for figures of seven, eight, nine, ten, eleven, and twelve sides severally?

4. How is it shewn, in the case of a circle described about a triangle, that if the triangle is acute-angled, the centre of the circle falls within it; if it is a right-angled triangle, the centre is in the side opposite to the right angle; and if it is an obtuse-angled triangle, the centre falls without the triangle beyond the side opposite to the obtuse angle?

5. Shew that a square described about a circle is double a square inscribed in the same circle.

6. State, in general terms, Euclid's method of describing about a circle an equilateral and equiangular polygon of any number of sides.

7. Shew that a hexagon can be made by placing together six equilateral triangles.

8. Apply the method of Prop. 16 to inscribe in a circle an equilateral and equiangular figure of twenty-four sides.

9. Shew that the centres of the circles inscribed in and described about an equilateral triangle coincide. Is the same necessarily true of equilateral figures having more than three sides?

10. If an equilateral and equiangular figure having any number of sides is inscribed in, or described about, a circle, shew how an equilateral and equiangular figure having twice as many sides can be inscribed in, or described about, the same circle.

11. Describe an equilateral and equiangular pentagon and hexagon on a given straight line.

12. Shew that in the figure of Proposition 10 there is a second triangle which has each of two of its angles double the third angle.

13. Shew that a square is the only right-angled parallelogram which can be described about a circle.

14. Apply Prop. 10 to divide a right angle into five equal parts.

15. Equilateral and equiangular figures of three, four, and six sides respectively are the only regular figures which can be placed together so as to fill up the space round a point.

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